TMM025 – Life Science Calculus II
(Updated 8 February 2021)

Suggested Number of Credit Hours: 3

This is the second course in a two-semester sequence of calculus courses intended for students majoring in the biological or environmental sciences and/or preparing for admission to medical, pharmaceutical, dental, veterinary, or other life-science-related professional schools. Students in this sequence must reason with limits, derivatives, integrals, and differential equations to describe and gain insight into biological processes and populations. Algebraic, logarithmic, exponential, and trigonometric functions are all used to model concepts from the life sciences. Questions from the life sciences should be used to motivate the content of the course, and the concepts and techniques taught should be used explicitly to answer those questions.

To qualify for TMM025 (Life Science Calculus II - LSCII), a course must achieve all the following essential learning outcomes listed in this document (marked with an asterisk). These make up the bulk of a Life Sciences Calculus II course. Courses that contain only the essential learning outcomes are acceptable from the TMM025 review and approval standpoint. It is up to individual institutions to determine further adaptation of additional course learning outcomes of their choice to support their students' needs. In addition, individual institutions will determine their own level of student engagement and manner of implementation. These guidelines simply seek to foster thinking in this direction. Sample tasks are listed to help clarify the intention of the essential learning outcomes, but no specific sample task is required for approval. Institutions are encouraged to tailor specific applications to the population of life science students at their institution.

In a Life Science Calculus II (TMM025) course, students should:

- develop effective thinking and communication skills;
- operate at a high level of detail;
- state problems carefully, articulate assumptions, understand the importance of precise definition, and reason logically to conclusions;
- identify and model essential features of a complex situation, modify models as necessary for tractability, and draw useful conclusions;
- deduce general principles from particular instances;
- use and compare analytical, visual, and numerical perspectives in exploring mathematics;
- assess the correctness of solutions, create and explore examples, carry out mathematical experiments, and devise and test conjectures;
- recognize and make mathematically rigorous arguments;
- read mathematics with understanding;
- communicate mathematical ideas clearly and coherently both verbally and in writing to audiences of varying mathematical sophistication;
- approach mathematical problems with curiosity and creativity, persist in the face of difficulties, and work creatively and self-sufficiently with mathematics;
- learn to link applications and theory;
- learn to use technological tools; and

- develop mathematical independence and experience open-ended inquiry.

– Adapted from the MAA/CUPM 2015 Curriculum Guide

1. Modeling with trigonometric Functions: Successful LSCII students understand how to model periodic phenomena by the use of trigonometric functions. They expand their knowledge of the concepts and rules of differential and integral calculus to trigonometric functions.

The successful LSCII student can:

1.1. Explain the structure of the graphs of $f(x) = \sin(x)$, $g(x) = \cos(x)$ (amplitude, period, horizontal and vertical intercepts) using the unit circle definition of the functions. *

1.2. Explain the role of the parameters A, B, C and D in the expressions $f(x) = A\sin(Bx + C) + D$ and $g(x) = A\cos(Bx + C) + D$. *

1.3. Recognize periodic phenomena and recognize that such phenomena can be modeled by functions of the form $f(x) = A_1\cos(B_1x + C_1) + A_2\sin(B_2x + C_2)$. *

Sample Tasks:

- The student determines the period and the amplitude of a function from a graph or a data set.
- Given a function which is defined on a bounded interval, the student defines and graphs its periodic extension.
- The student uses technology to explore the roles of the parameters of a function such as $f(x) = A\sin(Bx) + \cos(x)$.

1.4. Use technology to model given periodic data by finding values of the parameters in the expression $f(x) = A_1\cos(B_1x + C_1) + A_2\sin(B_2x + C_2)$. *

2. Calculus of Trigonometric Functions: Successful LSCII students can compute derivatives and integrals of trigonometric functions and interpret the results in context.

The successful LSCII student can:

2.1. Apply the concepts and rules of derivatives to functions to the basic trigonometric functions. *

Sample Tasks:

- The student uses a graph of $f(x) = \sin(x)$ to estimate the slope of a tangent line and approximate the derivative of $f(x)$ at a given point.
- The student computes the derivative of $y = \tan(x)$ using the quotient rule.
- Given a sinusoidal function model of the circadian rhythm for the body temperature of a mammal, the student finds and plots the derivative and interprets the plot.
- The student computes the extrema of a damped oscillation function.

2.2. Apply the concepts of integral calculus to the basic trigonometric functions. *

Sample Task:

- The student integrates y=tan(x) by using substitution.

3. Applications of Definite Integrals: Successful LSCII students can identify a definite integral of a function in terms of areas of regions between the graph of the function and the x-axis and use definite integrals to calculate areas of bounded regions. Students interpret a definite integral as an accumulation of change and apply this understanding to the life science calculus setting.

The successful LSCII student can:

3.1. Calculate area of bounded regions. *
Sample Tasks:

- The student symbolizes measurement of the area between a curve and the x-axis with a single definite integral or a sum or difference of definite integrals.
- The student symbolizes measurement of the area between two curves with a single definite integral or a sum or difference of definite integrals.
- The student calculates area measurement using the Fundamental Theorem of Calculus.

3.2. Interpret a definite integral as an accumulation of change and apply this understanding to the life science calculus setting. *
Sample Tasks:

- When given an applied accumulated change problem with relevance to the life sciences, the student recognizes that rate of change is known and accumulated change is being sought.
- The student symbolizes accumulated change with an appropriate definite integral.
- The student calculates accumulated change using the Fundamental Theorem of Calculus and states the appropriate units.
- The student interprets the meaning of a definite integral equation in the life science context. Example: "What is the meaning of the statement $\int_0^{10} \frac{dh}{dt}\, dt = 6$, if $\frac{dh}{dt}$ represents the growth rate of a tree and $h(t)$ is the height measured in feet and t is the time in years?"
- Using a sinusoidal model for the rate of airflow into the lungs, the student calculates the total amount of air inhaled in one cycle.

4. Integration Techniques: Successful LSCII students extend their knowledge of integration to find indefinite and definite integrals using integration by parts and partial fraction decomposition.

The successful LSCII student can:

4.1. Find indefinite and definite integrals using the method of integration by parts. *
Sample Tasks:

- Given an integral, the student identifies integration by parts as an appropriate technique.

- The student separates the integrand into two factors and correctly assigns each factor to the appropriate part of the integration by parts formula.
- The student applies the integration by parts formula and finds the correct integral.
- The student applies integration by parts on expressions which require multiple iterations of the method of integration by parts.
- The student solves life science application problems that require the method of integration by parts.

4.2. Find indefinite and definite integrals using the method of partial fraction decomposition. *

Sample Tasks:

- Given an integral, the student identifies partial fraction decomposition as an appropriate integration technique.
- The student decomposes a rational function into the sum of a polynomial and partial fractions.
- The student applies known formulas and techniques to integrate the decomposed form of the function.
- The student solves life science application problems that require the method of partial fraction decomposition.

5. Differential Equations: The successful student understands a differential equation as the mathematical model of a situation in which a quantity and its rate of change are dependent on one another. Students are able to use analytical as well as graphical methods. They can describe the behavior of solutions and interpret it in connection with physical situations. They can apply their understanding to a variety of life science problems.

The successful LSCII student can:

5.1. Solve separable differential equations by analytical methods. *
Sample Tasks:

- The student gives examples of differential equations.
- The student decides if a given function is a solution of a differential equation.
- Given the general solution of a differential equation and an initial condition, the student finds the particular solution of the initial value problem.
- The student decides if a given differential equation is separable.
- The student describes the steps of solving a differential equation using separation of variables.
- The student solves a first-order differential equation using separation of variables.
- The student analyzes the end behavior of a solution by computing a limit at infinity.

5.2. Understand the relationship between slope fields and solution curves of first-order differential equations. *
Sample Tasks:

- The student constructs a slope field associated with a first-order differential equation.

- The student matches a slope field to a given first-order differential equation.
- The student uses a slope field and an initial condition to approximate a solution curve of a differential equation.

5.3. Use a differential equation to interpret a physical situation in which a quantity and its rate of change are dependent on one another and apply this understanding to various life science problems. *

Sample Tasks:

- Given a physical situation described in words, the student models it by a differential equation or an initial value problem.
- The student expresses constant relative growth rate of a population by a differential equation.
- The student formulates a differential equation to represent population growth under external influences (such as emigration or harvesting).
- The student models population growth by a logistic equation.
- The student uses a linear differential equation to describe a cooling process.
- The student represents a mixing process (such as a continuous infusion, or pollution of a body of water) by a linear differential equation.
- The student explains the differences between an initial value problem modeling an infusion and an initial value problem modeling a single injection.
- The student critiques the reasonableness of a differential equation model by analyzing the qualitative behavior of the solutions.
- The student discusses the reasonableness of a model by analyzing the end behavior of the solutions.

5.4. Perform a qualitative analysis of an autonomous differential equation, without computing analytical solutions. *

Sample Tasks:

- The student determines the equilibrium solutions of a given autonomous differential equation using analytical methods.
- The student identifies the equilibrium solutions from a graph of $\frac{dy}{dx}$ as a function of y.
- The student analyzes a slope field to identify the equilibrium solutions, and to classify their stability.
- The student constructs a phase line and uses it to determine the stability of the equilibrium solutions.
- The student uses the stability behavior of the equilibrium solutions to sketch typical solution curves.
- The student predicts the end behavior of particular solutions by analyzing the stability of the equilibrium solutions.