

TMM021: Mathematics in Elementary Education I

(updated May 8, 2020)

Typical Range: 4-5 Semester Hours

Recommendation:

It is essential for all teachers of mathematics to understand the reasoning underlying the mathematics they are teaching. They need to understand why various procedures work, how each idea they will be teaching connects with other important ideas in mathematics, and how these ideas develop and become more sophisticated. Furthermore, knowing only the mathematics of the elementary grades is not sufficient to be an effective teacher of elementary grades mathematics. Neither is it sufficient to require that future teachers simply “take more math courses.” This document describes the kinds of mathematics and mathematical experiences that we believe are essential for their mathematical learning and professional development. Exploration in middle grade topics are introduced at the elementary level to begin planting the seed in preparation for advanced teachings.

We take the view that mathematics courses for future teachers must prepare them to do a different kind of work in their mathematics teaching than they likely experienced in their own schooling. With this in mind, we encourage oral and written communication in these classes as both a learning tool and as preparation for handling mathematical questions which arise in their classrooms. For example, we aim for discussion that focuses on the deep mathematical reasoning underlying the computational procedures that are usually taught in elementary school. We recommend this be done by exploring common misconceptions with preservice teachers and by engaging future teachers in activities that require them to interpret their own multiple ways of addressing questions and interpreting children’s work which might be incorrect, incomplete, or different from their adult ways of thinking. We aim to encourage a serious approach to the mathematics through questions (by instructors *and* preservice teachers) about *why* we do things the way we do, what the operations *mean*, what the *units* are on the answers, and how the mathematical ideas of the day *connect* to other mathematical ideas (looking for the “big ideas” in a problem).

We recommend these courses be activity based so that opportunities for deep, connected learning arise while misconceptions are addressed. This requires “good” problems and “good” questioning by instructors. A good problem needs to engage the preservice teachers at an appropriate level of challenge (hard enough that the preservice teacher cannot answer on autopilot). Often this is accomplished by confronting them with misconceptions framed as “a child said this...” and directions to analyze and/or justify the result. The justifications can themselves become a source of deep discussion - one preservice teacher may not understand another’s solution or explanation, a preservice teacher’s correct answer may have a flawed explanation, or a new method may be generated once an array of other methods has been shared. Sometimes the discussion is generated when an instructor says, “I want to list all of the different answers we got before we discuss the reasoning” (which means the class can actually discuss their own wrong answers). All of the above takes a lot of time, so we recommend 8 to 10 credit hours of such work in teacher education programs or prerequisites.

At the same time, we strongly urge that instructors be aware that these are college level mathematics courses. “A credit-bearing, college-level course in Mathematics must use the standards required for high school graduation by the State of Ohio as a basis and must do at least one of the following: 1) broaden, or 2) deepen, or 3) extend the student’s learning.” We recommend dedicated coursework for this content because there is much deep mathematics to be explored in understanding what we loosely term “elementary school mathematics.” This is a mathematical content course. The learning outcomes are all focused on using, justifying, and connecting mathematical concepts and do not address “how to

teach.” It is sometimes appropriate to discuss topics which are more directly relevant to a methods course when they serve the purpose of motivating a mathematical discussion, but students should not be assessed on methods in these courses.

The courses should integrate reasoning, flexibility, multiple explanations, and number sense. Leading questions help students to make connections among topics and to develop their own questioning skills (e.g. Have we seen this idea before? How are these two different solution methods related to each other? What does it mean? How do you know? Could you draw a picture to show it? Where did they go wrong? Could we use their idea in this other problem?) Students should understand that mathematics is correct if it makes personal sense and if one can explain it in a way to make sense to others, not if an authority certifies it. Preservice teachers should leave these courses knowing that math makes sense and armed with the underlying knowledge they need to make math make sense for their future students. Elementary students will often come up with their own creative approaches to problems, so future teachers must be able to evaluate their mathematical viability **before** deciding how to respond instructionally. Squashing a child’s idea can be quite harmful. And often children’s ideas are right or almost right.

The learning outcomes that follow are all viewed as essential by the committee (marked with an asterisk) and demonstrate the level of student engagement motivating this course. In addition, learning outcomes are not specific items/topics but rather learning outcomes of course entirety. Institutional courses should provide an integrated experience with learning outcomes woven together throughout the course.

1. Numbers

The successful Mathematics in Elementary Education student can:

1a. Discuss the intricacies of learning to count, including the distinction between counting as a list of numbers in order and counting to determine a number of objects, and use pairings between elements of two sets to establish equality or inequalities of cardinalities. *

1b. Attend closely to units (e.g., apples, cups, inches, etc.) while solving problems and explaining solutions. *

1c. Discuss how the base-ten place value system (including extending to decimals) relies on repeated bundling in groups of ten and how to use objects, drawings, layered place value cards, base-ten blocks, and numerical expressions (including integer exponents) to help reveal base-ten structure. *

1d. Use the CCSS (Common Core State Standards) development of fractions: *

- Start with a whole.
- Understand the fraction $1/b$ as one piece when the whole is divided into b equal pieces.
- Understand the fraction a/b as a pieces of size $1/b$ and that the fraction a/b may be larger than one.
- Understand fractions as numbers that can be represented in a variety of ways, such as with lengths (esp. number lines), areas (esp. rectangles), and sets (such as a collection of marbles).
- Use the meaning of fractions to explain when two fractions are equivalent.

1e. Model positive versus negative numbers on the number line and in real-world contexts. *

1f. Reason about the comparison ($=$, $<$, $>$) of numbers across different representations (such as fractions, decimals, mixed numbers, ...). *

1g. Demonstrate the skill of calculating simple arithmetic problems WITHOUT the use of a calculator. *

2. Operations

The successful Mathematics in Elementary Education student can:

2a. Recognize addition, subtraction, multiplication, and division as descriptions of certain types of reasoning and correctly use the language and notation of these operations. *

2b. Illustrate how different problems are solved by addition, subtraction, multiplication and division and be able to explain how the operation used is connected to the solving of the problem. *

2c. Recognize that addition, subtraction, multiplication, and division problem types and associated meanings for the operations (e.g., CCSS, pp. 88–89) extend from whole numbers to fractions and decimals. *

2d. Employ teaching/learning paths for single-digit addition and associated subtraction and single-digit multiplication and associated division, including the use of properties of operations (i.e., the field axioms). *

2e. Compare and contrast standard algorithms for operations on multi-digit whole numbers that rely on the use of place-value units (e.g., ones, tens, hundreds, etc.) with mental math methods students generate. *

2f. Use math drawings and manipulative materials to reveal, discuss, and explain the rationale behind computation methods. *

2g. Extend algorithms and mental math methods to decimal arithmetic. *

2h. Use different representations of the same fraction (e.g., area models, tape diagrams) to explain procedures for adding, subtracting, multiplying, and dividing fractions. (This includes connections to grades 6–8 mathematics). *

2i. Explain the connection between fractions and division, $a/b = a \div b$, and how fractions, ratios, and rates are connected via unit rates. (This includes connections to grades 6–8 mathematics. See the Ratios and Proportional Relationships Progression for a discussion of unit rate). *

2j. Explain why the extensions of the operations to signed numbers make sense. *

3. Algebraic Thinking

The successful Mathematics in Elementary Education student can:

3a. Model and communicate their reasoning about quantities and the relationships between quantities using a variety of representations. *

3b. Discuss the foundations of algebra in elementary mathematics, including understanding the equal sign as meaning “is the same [amount] as” rather than a “calculate the answer” symbol. *

3c. Look for regularity in repeated reasoning, describe the regularity in words, and represent it using diagrams and symbols and communicate the connections among these. *

3d. Articulate, justify, identify, and use properties of operations. *

3e. Describe numerical and algebraic expressions in words, parsing them into their component parts, and interpreting the components in terms of a context. *

3f. Use a variety of methods (such as guess and check, pan balances, strip diagrams, and properties of operations) to solve equations that arise in “real-world” contexts. *

4. Number Theory

The successful Mathematics in Elementary Education student can:

4a. Demonstrate knowledge of prime and composite numbers, divisibility rules, least common multiple, greatest common factor, and the uniqueness (up to order) of prime factorization. *

4b. Discuss decimal representation and recognize that there are numbers beyond integers and rational numbers. *