TMM022: Mathematics in Elementary Education II
(updated May 8, 2020)

Typical Range: 4-5 Semester Hours

Recommendation:
It is essential for all teachers of mathematics to understand the reasoning underlying the mathematics they are teaching. They need to understand why various procedures work, how each idea they will be teaching connects with other important ideas in mathematics, and how these ideas develop and become more sophisticated. Furthermore, knowing only the mathematics of the elementary grades is not sufficient to be an effective teacher of elementary grades mathematics. Neither is it sufficient to require that future teachers simply "take more math courses." This document describes the kinds of mathematics and mathematical experiences that we believe are essential for their mathematical learning and professional development. Exploration in middle grade topics are introduced at the elementary level to begin planting the seed in preparation for advanced teachings.

We take the view that mathematics courses for future teachers must prepare them to do a different kind of work in their mathematics teaching than they likely experienced in their own schooling. With this in mind, we encourage oral and written communication in these classes as both a learning tool and as preparation for handling mathematical questions which arise in their classrooms. For example, we aim for discussion that focuses on the deep mathematical reasoning underlying the computational procedures that are usually taught in elementary school. We recommend this be done by exploring common misconceptions with preservice teachers and by engaging future teachers in activities that require them to interpret their own multiple ways of addressing questions and interpreting children's work which might be incorrect, incomplete, or different from their adult ways of thinking. We aim to encourage a serious approach to the mathematics through questions (by instructors *and* preservice teachers) about *why* we do things the way we do, what the operations *mean*, what the *units* are on the answers, and how the mathematical ideas of the day *connect* to other mathematical ideas (looking for the "big ideas" in a problem).

We recommend these courses be activity based so that opportunities for deep, connected learning arise while misconceptions are addressed. This requires "good" problems and "good" questioning by instructors. A good problem needs to engage the preservice teachers at an appropriate level of challenge (hard enough that the preservice teacher cannot answer on autopilot). Often this is accomplished by confronting them with misconceptions framed as "a child said this…" and directions to analyze and/or justify the result. The justifications can themselves become a source of deep discussion - one preservice teacher may not understand another's solution or explanation, a preservice teacher's correct answer may have a flawed explanation, or a new method may be generated once an array of other methods has been shared. Sometimes the discussion is generated when an instructor says, "I want to list all of the different answers we got before we discuss the reasoning" (which means the class can actually discuss their own wrong answers). All of the above takes a lot of time, so we recommend 8 to 10 credit hours of such work in teacher education programs or prerequisites.

At the same time, we strongly urge that instructors be aware that these are college level mathematics courses. "A credit-bearing, college-level course in Mathematics must use the standards required for high school graduation by the State of Ohio as a basis and must do at least one of the following: 1) broaden, or 2) deepen, or 3) extend the student's learning." We recommend dedicated coursework for this content because there is much deep mathematics to be explored in understanding what we loosely term "elementary school mathematics." This is a mathematical content course. The learning outcomes are all focused on using, justifying, and connecting mathematical concepts and do not address "how to

teach." It is sometimes appropriate to discuss topics which are more directly relevant to a methods course when they serve the purpose of motivating a mathematical discussion, but students should not be assessed on methods in these courses.

The courses should integrate reasoning, flexibility, multiple explanations, and number sense. Leading questions help students to make connections among topics and to develop their own questioning skills (e.g. Have we seen this idea before? How are these two different solution methods related to each other? What does it mean? How do you know? Could you draw a picture to show it? Where did they go wrong? Could we use their idea in this other problem?) Students should understand that mathematics is correct if it makes personal sense and if one can explain it in a way to make sense to others, not if an authority certifies it. Preservice teachers should leave these courses knowing that math makes sense and armed with the underlying knowledge they need to make math make sense for their future students. Elementary students will often come up with their own creative approaches to problems, so future teachers must be able to evaluate their mathematical viability *before* deciding how to respond instructionally. Squashing a child's idea can be quite harmful. And often children's ideas are right or almost right.

The learning outcomes that follow are all viewed as essential by the committee (marked with an asterisk) and demonstrate the level of student engagement motivating this course. In addition, learning outcomes are not specific items/topics but rather learning outcomes of course entirety. Institutional courses should provide an integrated experience with learning outcomes woven together throughout the course.

1. **Ratios, Proportional Relationships, and Functions** (this has connections to Grades 6-8)
   The successful Mathematics in Elementary Education student can:

   1a. Reason about how quantities vary together in a proportional relationship, using tables, double number lines, and tape diagrams as supports. *

   1b. Distinguish proportional relationships from other relationships, such as additive relationships and inversely proportional relationships. *

   1c. Use unit rates to solve problems and to formulate equations for proportional relationships (see measurement). *

   1d. Recognize that unit rates make connections with prior learning by connecting ratios to fractions. *

   1e. View the concept of proportional relationship as an intellectual precursor and key example of a linear relationship. *

   1f. Examine and reason about functional relationships represented using tables, graphs, equations, and descriptions of functions in words. In particular, students can examine the way two quantities change together using a table, graph, and equation. *

   1g. Examine the patterns of change in proportional and linear relationships and the types of real-world situations these functions can model and contrast with nonlinear relationships. *

2. **Measurement**
   The successful Mathematics in Elementary Education student can:

   2a. Explain the general principles of measurement, the process of iterations, and the central role of units (including nonstandard, U.S. customary, and metric units). *

   2b. Explain how the number line connects measurement with number through length. *

   2c. Understand and distinguish area and volume, giving rationales for area and volume formulas that can be obtained by finitely many compositions and decompositions of unit squares or unit cubes, including but not limited to formulas for the areas of rectangles, triangles, and parallelograms, and volumes of arbitrary right prisms. (This includes connections to grades 6–8 geometry, see the Geometric Measurement Progression.). *

   2d. Describe how length, area, and volume of figures change under scaling, focusing on areas of parallelograms and triangles, with counting-number scale factors. *

   2e. Informally develop the formulas for area and circumference of a circle and use them in solving real-world problems. *

   2f. Attend to precision in measurement with rounding guided by the context. *

   2g. Convert between different units both by reasoning about the meaning of multiplication and division and through dimensional analysis. *

3. **Geometry**
   The successful Mathematics in Elementary Education student can:

   3a. Understand geometric concepts of angle, parallel, and perpendicular, and use them in describing and defining shapes. *

   3b. Describe and reason about spatial locations (including the coordinate plane). *

   3c. Informally prove and explain theorems about angles and solve problems about angle relationships. *

   3d. Classify shapes into categories and reason to explain relationships among the categories. *

   3e. Explain when and why the Pythagorean Theorem is valid and use the Pythagorean Theorem in a variety of contexts. *

   3f. Examine, predict, and identify translations, rotations, reflections, and dilations, and combinations of these. *

   3g. Understand congruence in terms of translations, rotations, and reflections; and similarity in terms of translations, rotations, reflections, and dilations and solve problems involving congruence and similarity. *

3h. Understand symmetry as transformations that map a figure onto itself. *

4. Statistics and Probability
The successful Mathematics in Elementary Education student can:

4a. Recognize and formulate a statistical question as one that anticipates variability and can be answered with data. *

4b. Understand various ways to summarize, describe, and compare distributions of numerical data in terms of shape, center (e.g., mean, median), and spread (e.g., range, interquartile range). *

4c. Use measures and data displays to ask and answer questions about data and to compare data sets. (This includes connections to grades 6–8 statistics.). *

4d. Distinguish categorical from numerical data and select appropriate data displays. *

4e. Use reasoning about proportional relationships to argue informally from a sample to a population. *

4f. Calculate theoretical and experimental probabilities of simple and compound events, and understand why their values may differ for a given event in a particular experimental situation. *

4g. Explore relationships between two variables by studying patterns in bivariate data. *