TMM014 – TECHNICAL MATHEMATICS I
(updated May 6, 2020)

Typical Range: 4-5 Semester Hours

A course in Technical Mathematics specializes in the application of mathematics to the engineering technologies. The course emphasizes critical thinking by placing students in problem-solving situations and supporting students as they learn to make decisions, carry out plans, and judge results. Students encounter contextualized situations where concepts and skills associated with measurement, algebra, geometry, trigonometry, and vectors are the pertinent tools. The course highlights the supporting algebraic and analytical skills.

As a mathematics course in the applied fields, students studying Technical Mathematics 1 (TMM014) will benefit from more active and collaborative learning. Instead of extensive lectures dominating the presentation of skills and procedures, we hope this course will place students in situations where the mathematics become the active tools for investigation. Applications should be the foundation of a collaborative experience, where groups of students make decisions, choose tools, follow plans, draw conclusions, and explain their reasoning.

To qualify for TMM014 (Technical Mathematics 1), a course must achieve all of the following essential learning outcomes listed in this document (marked with an asterisk). The Illustrations exemplify the level of student engagement motivating this course.

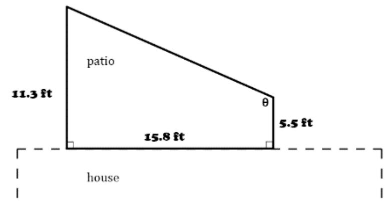
1. Geometry: Successful Technical Mathematics students are able to visualize and identify geometric aspects and relationships within a given situation. Students can use these relationships to disseminate measurements throughout the situation. By reasoning with geometric properties, students can target specific information and explain their thought process.

The successful Technical Mathematics student can:

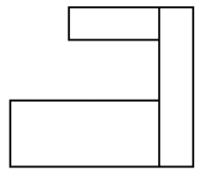
1a. recognize and apply properties of 2D shapes. Students recognize shapes within diagrams and use shapes in creating representations of situations. By selecting appropriate aspects and characteristics of the geometric shapes, students can extend a situational description via related lengths and areas. Through similar shapes and classifications, students explain their approach to problem solving. *

Illustrations:

- A house is replacing their backyard concrete patio. Here is an overhead view. The patio is to be 6 $inches$ deep. Calculate how many cubic yards of concrete are needed.

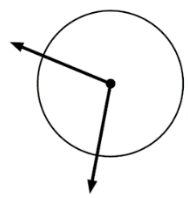


- Here is a rectangular-ish looking object. Use a ruler to determine its area.

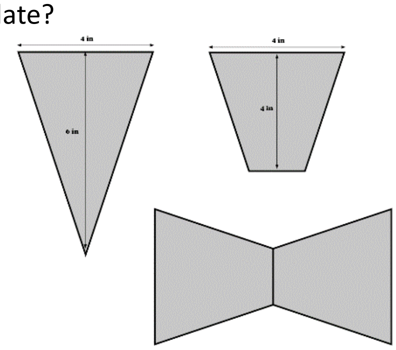


Somewhere you can make a horizontal cut and cut this object in half, which creates two objects of equal area. How far from the bottom should you make the horizontal cut to cut the object in half? Explain, with some calculations, how you decided on the location? (Don't just eyeball it.)

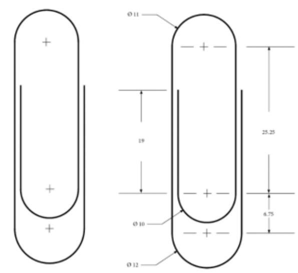
- Using a ruler and protractor, determine the circumference of the circle and the lengths of both subtended arcs.



- An eighth-inch brass bowtie plate is fashioned by truncating a $6\ in\ \times\ 4\ in$ triangle at a height of $4\ in$ and then welding the cross sections together. What is the shortest (middle) distance across this bowtie plate?



- Here is a schematic of a paperclip (measurements are given in mm). Determine the length of wire need to create the paperclip, to two decimal places. (Ø stands for diameter.)



1b. recognize and apply properties of 3D shapes. Students visualize 3D situations and navigate around the picture or diagram. Through the use of common 3D shapes, students calculate volumes need for further calculations. *

Illustrations:

- A rectangular bar of gold measures $15\ cm. \times 7\ cm. \times 5\ cm.$ What is the volume of the gold bar? Look up the density of gold and describe with a rate equation. What is the weight of the gold bar?
- A conductor material has a free-electron density of $10^{24}\ \frac{electrons}{m^3}$. How many electrons are there in $1\ cm^3$?
- Stress is defined as force over area: $\sigma = \frac{P}{A}$. For a cylindrical bar, the formula for stress is given by

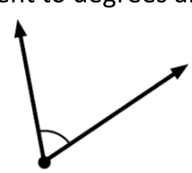
$$\sigma = \frac{P}{\pi r^2}$$

a) Solve this for the radius, r.
b) If the radius is tripled, then how exactly is the value of stress affected?

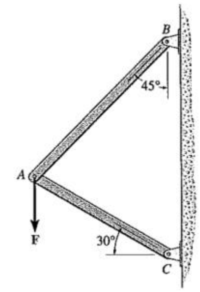
1c. recognize and apply properties of angles. Student measures angles in degrees, radians, and DMS. Angular relationships, together with obtained or given angular measurements, are combined by student to elaborate about further angular measurements. *

Illustrations:

- Using a protractor, obtain the measurement of the angle below to the nearest tenth of a degree. Then convert this measurement to degrees and DMS.

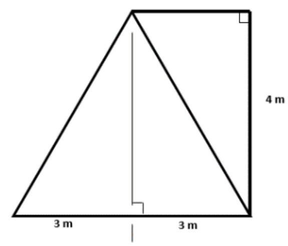


- Two steel bars are attached to a vertical wall as well as to each other as seen in the diagram below. $F = 350N$ and points straight down.

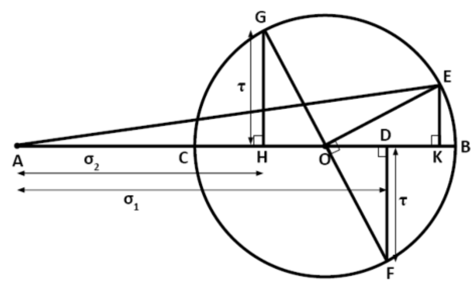


  a) The force F induces a force in each bar. On the diagram draw the direction of the force in each bar.
  b) Determine the magnitude of the force in each bar.

- Just by calculating (no rulers), determine all sides lengths and all angles of the following truss



- Another version of Mohr's circle includes shear stress, τ.



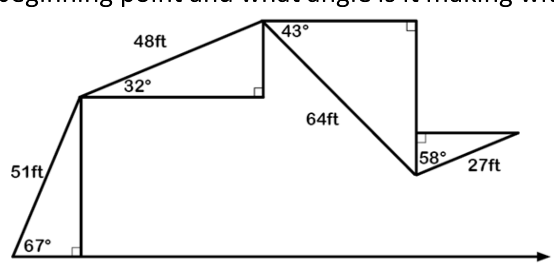
Suppose $\sigma_1 = 65\ \frac{N}{mm^2}$, $\sigma_2 = 35\ \frac{N}{mm^2}$,
$\tau = 25\frac{N}{mm^2}$.

Determine all lengths and angles.

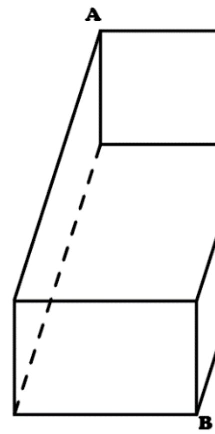
1d. apply the Pythagorean Theorem. The student can identify right angle triangles in a diagram and attribute given measurements to the proper aspects. With this information, the student can use the Pythagorean Theorem to deduce other measurements. *

Illustrations:

- A surveyor has diagramed her path as she traverses a proposed building site. How far is her final position from the beginning point and what angle is it making with the East direction.



- A 3-dimensional rectangular prism has dimensions $3\ in$, $5\ in$, and $9\ in$. Determine the exact length of the between corner A and B, the corners farthest apart.



2. Measurement: Successful Technical Mathematics students are comfortable with measurements and their representations. Students anticipate types and classifications of measurements and use their expectations to formulate lines of investigation. Students obtain, convert, compare, and combine measurements. They express measurements in a variety of ways.

The successful Technical Mathematics student can:

2a. reason with amount measurements. Students distinguish qualities, characteristics, and aspects of objects in situations that can be measured. They connect units with appropriate measurement type. They convert units, and abbreviations, for similar types of measurements and can properly use units when discussing measurements. *

Illustrations:

- There are _________ Pascals in 1 pound per square inch.
- What do each of the units below measure? Also, give the abbreviation for each unit. Pound, Liter, Newton, Pascal, Cubic Millimeter, Volt, Coulomb, Inch, Foot, Second, Minute, Celsius Degree, Joule, Square meters, Gallon, Hour, Watt, Horsepower, Gram, Kilometer, Ampere, Calorie, Millimeter, Square Feet, Milliliter, BTU, Fahrenheit Degree, Cubic Feet, Square Inch, Mile, Ohm, Farad, Centimeter, Atmosphere, Acre
- Let a metallic conductor having a resistance of R_0 at 0°C be heated to t°C. Let its resistance at this temperature be R_t. Then, $R_t = R_0\, \alpha\, t + R_0$ is a linear function in t. α is called the *temperature coefficient of resistance* of the conductor. For copper, $\alpha = \frac{1}{234.5}$ (units?) What are the units of α? (Remember, the units of resistance are Ohms, Ω.)

2d. communicate about measurements. Students communicate fluently with units, abbreviations, and notation whether this be verbally or in writing. They calculate accurately with decimals, fractions, percentages, scientific notation, and engineering notation. They do so with information they have deciphered from tables and graphs. *

Illustrations:

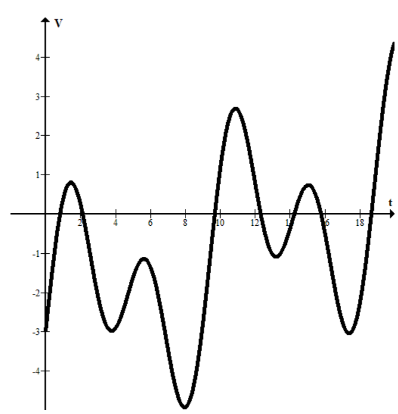
- A conductor material has a free-electron density of $10^{24}\ \frac{electrons}{m^3}$. When a voltage is applied, a constant drift velocity of $1.5 \times 10^{-6}\ \frac{m}{sec}$ is attained by the electrons. How far have the electrons drifted down the conductor in 2 seconds? Express your answer in engineering notation.

- The heat capacity of kerosene is $0.48 \frac{Btu}{lb \cdot °F}$. Is the lb in the denominator a variable waiting for a value to be substituted in?
- When a bar of length L and cross-section A and made of a metal with a Young's Modulus of E is subjected to a load of P, then it stretches a length δ called the deformation, deflection, or elongation. These are related by

$$\delta = \frac{P \cdot L}{A \cdot E}$$

a) If load on a particular rod is increased, then how does the elongation change.
b) If a thicker bar of the same length and material is subjected to the same load, then how will the deformation be affected?
c) If the cross-sectional area is doubled, then how is the elongation affected?
d) If both the load and the cross-sectional area are doubled, then how is the elongation affected?

- The graph of the voltage across a terminal in a circuit is graphed below.



Circle a positive or negative change for voltage during each second below, or neither.

Time in Seconds		Voltage Change		
from	to	positive	negative	neither
0	1	positive	negative	neither
1	2	positive	negative	neither
2	3	positive	negative	neither
3	4	positive	negative	neither

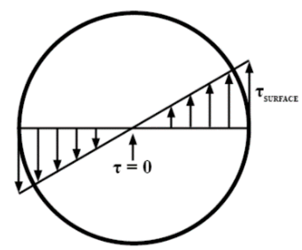
3. Equations & Graphs: Successful Technical Mathematics students are proficient at algebraic procedures and manipulating measurements via equations and graphs. Students can express dependence and independence via equations and calculate resulting measurements via formulas. These calculations might be numeric, but more than likely, students should feel comfortable with the algebra of measurements.

The successful Technical Mathematics student can:

3a. algebraically manipulate expressions and equations. Students are fluent in the use of arithmetic and algebra rules, especially fractions. They predict effects of changes in variables. They solve equations for dependent variables. They model situations with linear and quadratic equations. *

Illustrations:

- Solve the formula for tensile loads, $\delta = \frac{P \cdot L}{E \cdot A}$, for E.
- Many machine parts, like a driveshaft, are loaded in torsion to transmit power. Apply a torque, T, and the shaft will twist through an angle, θ. Twisting means the material is deforming, so we have strain in the material.
  The greatest strain is at the surface, while the strain is 0 at the center. The strain varies <u>linearly</u> from the center to the surface of the shaft.

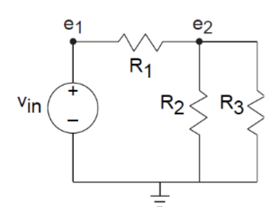


  On a shaft of diameter $0.5\ in$, the shear stress on the surface is $18.7\ ksi$. What is the stress at a radius of $0.18\ in$? Convert to MPa.

- In the circuit below, the three resistors are connected at e$_2$, which is called a node.

  The voltage at node e$_2$ is given by

$$v_2 = \frac{R_2\ R_3}{R_1\ R_2 + R_1\ R_3 + R_2\ R_3}\ v_{in}$$



  Calculate the voltage at node e$_2$ when $v_{in} = 5.4V$, $R_1 = 2.1k\Omega$, $R_2 = 1.7k\Omega$, and $R_3 = 1.8k\Omega$

- Deflection due to thermal expansion is given by $\delta = \alpha \cdot L \cdot \Delta T$
  View the variables δ and α as directly proportional. According to the equation, what does the proportion constant equal?

- Power is measured in Watts. $1\ Watt = \frac{1\ Joule}{1\ sec}$. $\left(1W = \frac{1\ J}{1\ s}\right)$

  Suppose a hydro-electric station has a turbine and generator, which converts water potential/kinetic energy to electric energy. Suppose they deliver $40MW$ for $6\ hours$. How many joules is this output?

  This gives the output energy needed from the turbine and generator. Suppose the turbine has an 86% efficiency and the generator has an efficiency of 92%. Then what was the energy in joules supplied to the system by the water?

- The equation relating capacitance, charge, and voltage is

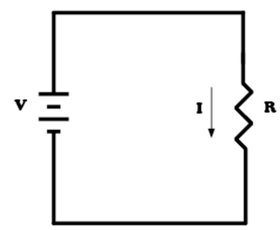
$$C = \frac{Q}{V}$$

a) If you increase the voltage and keep charge the same, what happens to the capacitance?
b) If you decrease the voltage and keep charge the same, what happens to the capacitance?
c) If you increase the charge and keep voltage the same, what happens to the capacitance?

3b. combine algebraic and graphical representations. Students are proficient with all representations used to describe information and measurements. In particular, students competently use lines and linear equations to describe real-world relationships. *

Illustrations:

- For a fixed resistor, Ohm's Law gives a linear equation, $V = I\,R$. Suppose we have a circuit with a fixed resistor. The R would be a constant. V and I would be the variables. Which would be the dependent and independent variable? Explain your decision.



  Suppose a particular circuit has a 3Ω resistor. What would the equation for voltage look like? Draw a graph for voltage.

- The nucleus of a helium atom consists of two protons and two neutrons. Two electrons orbit this nucleus. If the electrons are removed, then the remaining particle is called an alpha particle ($\alpha - particle$).

  The electrostatic force between two particles is given by

$$F = k\frac{Q_1\,Q_2}{d^2}$$

  Draw the general shape of the graph of electrostatic force vs. separating distance

- Thermal expansion is given by $\delta = \alpha\,L\,\Delta T$, where α is the thermal coefficient and ΔT is the change in temperature. For an aluminum bar of length $8\ inches$, the expansion is a linear function of the temperature change. What is the rate of change of expansion with respect to temperature change?

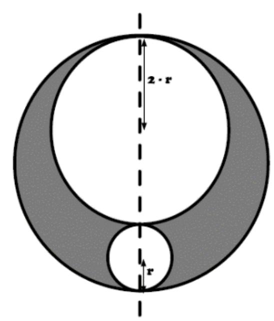
- As it turns out, the temperature coefficient of resistance of the conductor also depends slightly on the temperature.

Different values of α for copper: $\alpha(T)$							
Temp. in ℃	0	5	10	20	30	40	50
α	0.00427	0.00418	0.00409	0.00393	0.00378	0.00364	0.00352

a) Evaluate $\alpha(20℃)$
b) Evaluate $\alpha(25℃)$

- A diameter runs through a circle of radius 2. Move out a distance r along the diameter and draw an internal tangent circle of radius r. And, also draw a tangent circle of radius $(2 - r)$ along the same diameter. See diagram.

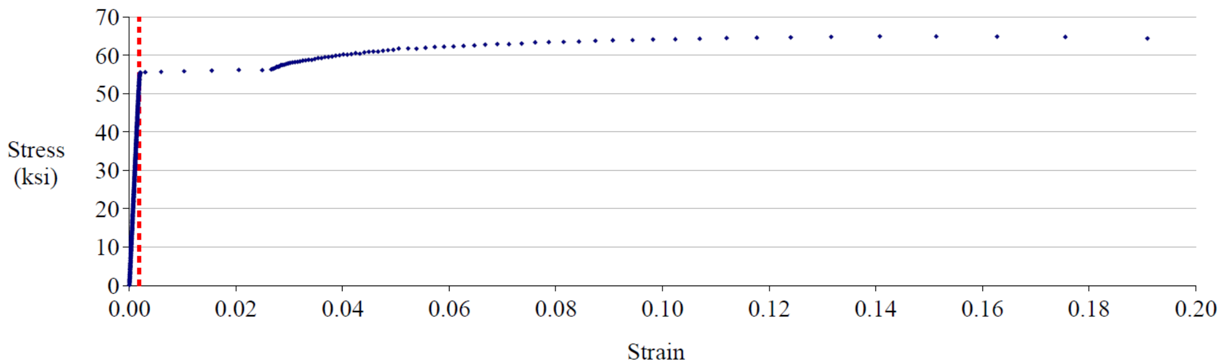
The outer circle has an area. When you remove the areas of the two inner circles, then you have the remaining area left over inside the outer circle. This leftover area changes if as you change r.



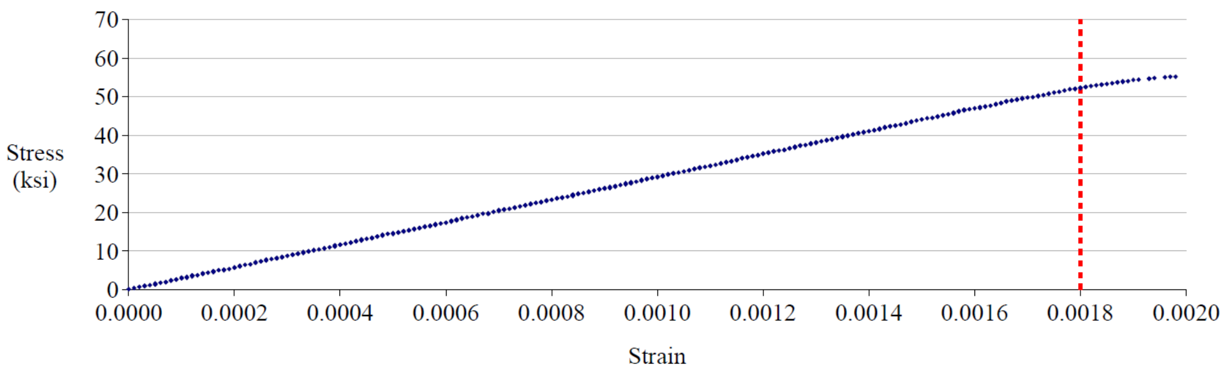
If $r = 0$, then the leftover area is 0.
If $r = 2$, then the leftover area is 0.
If $0 < r < 2$, then the leftover area varies.

a) Write a formula for this leftover area with respect to r.
b) Go to desmos.com and graph your formula.
c) Which value of r results in the greatest amount of leftover area?
d) What is the greatest amount of leftover area?

- Below is a stress-strain diagram for low-carbon steel.



Here is an enlargement of the extreme left part of the graph above. The two dotted vertical lines are the same line in each graph.



a) This straight part of the stress-strain graph is called the elastic region. What is the slope of the graph in the elastic region?
b) Create a formula for a linear function representing the elastic region.
Remember: σ is the symbol for stress and ϵ is the symbol for strain.
c) What is the rate of change of your linear function?
d) How do you interpret the rate of change in this situation?

- The strength of an electric field is given by $E = \frac{F}{Q}$, where F is the force (in Newtons) acting on a positive charge Q (in Coulombs C). If we fix a charge of $1.602 \times 10^{-19}\ C$, then the electric field strength is a linear function to force, in Newtons.

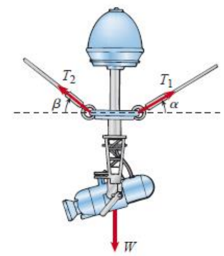
a) Write a formula for this situation
b) What would be the slope of the line graph for E?
c) What is the rate of change of the electric field strength with respect to the force?

3c. solve systems of equations. Students can create systems of linear equations from given situations and then solve them via various techniques. *

Illustrations:

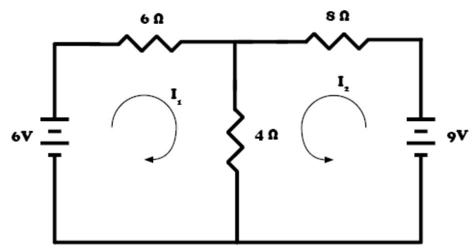
- A moveable overhead camera, used for close-up viewing of field action at sporting events, is supported by two steel wires. The camera weighs $W = 37\ lbs$ (force). Ignore the weight of the wires.

  At some time during a football game, the camera is positioned such that $\alpha = 21°$ and $\beta = 39°$. This causes tension (force) in the wires, which we can represent as vectors, T_1 and T_2.

  The camera is stationary, which means all three vectors add up to $\vec{0}$. Figure out, the values of T_1 and T_2.

- In the circuit below, there are two loops. Current I_1 loops clockwise in the left loop. Current I_2 loops counterclockwise in the right loop.

  If we travel around the left loop, then we begin with an increase of $6v$ from the battery. Then there is a voltage drop of $I_1 6$. Then there is a voltage drop of $(I_1 + I_2)4$. And, we are back to the battery.

  $$6 - I_1 6 - (I_1 + I_2)4 = 0$$

  If we travel around the right loop, then we begin with an increase of $9v$ from the battery. Then there is a voltage drop of $I_2 8$. Then there is a voltage drop of $(I_1 + I_2)4$. And, we are back to the battery.

  $$9 - I_2 8 - (I_1 + I_2)4 = 0$$

  Their graphs of these two equations would be two lines, which intersect in a single point. What is this point?

4. Dimensional Analysis: Successful Technical Mathematics students are proficient at combining measurements. Dimensions simply refers to products of measurements, which naturally emerge when working with rates. Students reason through the connecting changes described by the rates and manipulate units symbolically via exponential forms.

The successful Technical Mathematics student can:

4a. reason with rate measurements. Students express rates as phrases, equations, and fractions. They compare the changing amounts described by rates. They can convert units and reason for the needed rate. *

Illustrations:

- According to the Internet, $1\ BTU = 1055.06\ joules$. What would we multiply both sides by to get a conversion for $775\ joules$?
- The resistance in a wire conductor is given by $R = \rho\ \frac{L}{A}$.
  Where ρ is the resistivity, L is the length of the wire, and A is the cross-sectional area of the wire. Circle one:
  - R and ρ are INVERSELY/DIRECTLY proportional
  - R and L are INVERSELY/DIRECTLY proportional
  - R and A are INVERSELY/DIRECTLY proportional
  - L and A are INVERSELY/DIRECTLY proportional
- Above $79°F$ hydrogen cyanide becomes a colorless, toxic gas. A vial of hydrogen cyanide is currently at $68°F$, but is sitting near a furnace heating the room by $2°C$ per $5\ min$. How long before the basement becomes toxic?
- What does $\delta(A) = \frac{P \cdot L}{A \cdot E}$ convey about which measurements vary and which remain constant? Think up a situation where this might be appropriate.
- $1\ ampere = \frac{1\ Coulomb}{1\ sec}$ is a rate measurement. $1\ ampere$ of current means that $1\ Coulomb$ of electrons are flowing past a location every second. For a $2.7\ A$ current, how many electrons flow past a location every $1.5\ secs$?
- Describe the changing quantities indicated by the rate, “miles per hour per minute”.
- Describe the changing quantities indicated by the rate $\frac{4\ N}{3\ in \cdot 5°C}$.
- Describe the changing quantities indicated by the rate $\frac{\mu m / m}{°F}$.
- The heat capacity of kerosene is $0.48 \frac{Btu}{lb \cdot °F}$. If the temperature of $3\ lbs$ of kerosene is raised $15°F$, then what will be the corresponding change in Btu?
- We represent situational rates as phrases, equations, and fractions. One of these is given below. You supply the other two.
  a) $27\ N$ per $7\ in^2$
  b) $45\ MPa = 0.05\ cm$
  c) $\frac{30\ mg}{7\ mL}$

- Heat a piece of steel and it expands. Cool the same piece, and it shrinks. A metal pole of length L will lengthen by δ when the temperature rises. This change in length, δ, bar length, L, and change in temperature, ΔT, are related by

$$\delta = \alpha \cdot L \cdot \Delta T$$

$\alpha_{steel} = \frac{12\times10^{-6}}{°C}$ and $\alpha_{concrete} = \frac{11\times10^{-6}}{°C}$, which are basically equal. Why do you think steel bars are used for concrete reinforcement rather than aluminum bars?

4b. manipulate algebraic representations of amounts and rates. Students algebraically manipulate rates written as fractions. They can reduce and simplify products of rate fractions using the properties of exponents. *

Illustrations:

- Simplify $\frac{kg}{m \cdot s^2} \cdot \frac{s}{m^2 \cdot kg}$
- Explain the equivalence of $\frac{m^2 \cdot kg}{s^3 \cdot A}$ and $m^2 \cdot kg \cdot s^{-3} \cdot A^{-1}$
- List steps in the conversion of $\frac{s^4 \cdot A^2}{m^2 \cdot kg}$ to $\frac{C}{V}$
- Converts mi^2 to ft^2 .

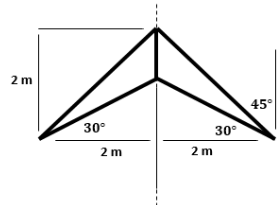
5. Trigonometry: Successful Technical Mathematics students demonstrate an ability to use triangles. Students generate a triangle mesh as needed to combine given information together and propagate a chain of measurements toward a desired target.

The successful Technical Mathematics student can:

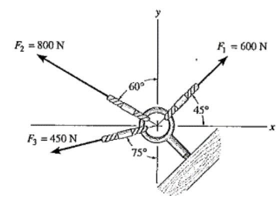
5a. identify right triangles in situations. Students identify right triangular configurations in diagrams and assign given measurements to the proper pieces of the triangles. *

Illustrations:

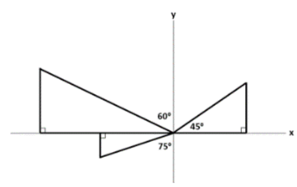
- Just by calculating (no rulers), determine all sides lengths and all angles of the following truss.



- An eyehook is screwed into a beam and three ropes are tied to it. The ropes are each experiencing a force as diagramed below.



We can create a diagram representing these forces as hypotenuses of right triangles.

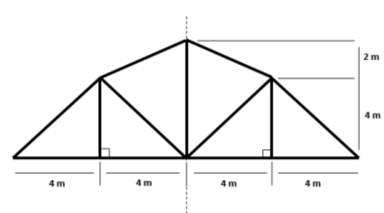


a) The length of each hypotenuse is the magnitude of the corresponding force. Add these to the diagram.
b) Determine all of the horizontal and vertical components of the forces.

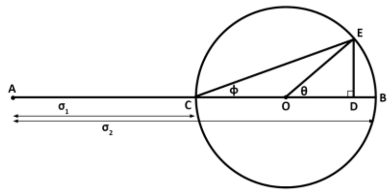
5b. apply trigonometric functions. Students identify which pieces of a right triangle are given and apply sine, cosine, and tangent functions to obtain further measurements. They use calculators to evaluate expressions involving trigonometric functions. They apply the Laws of Sine and Cosine appropriately. *

Illustrations:

- Just by calculating (no rulers), determine all sides lengths and all angles of the following truss



- Mohr's Circle is a graphical method of finding normal, tangential, and resultant stresses on an oblique plane. Here is the diagram when a body is subjected to two mutually perpendicular principal tensile stresses of unequal Intensities. The lengths inside the diagram represent magnitudes of various stresses. For example, the magnitude of stress σ_1 is represented by the length of the line segment $\overline{AC}$. Therefore, mm in the diagram represent $\frac{N}{mm^2}$. Unfortunately, the diagram is not to scale. You cannot simply measure the lengths with a ruler. It will take a bit of mathematical deduction and calculation.



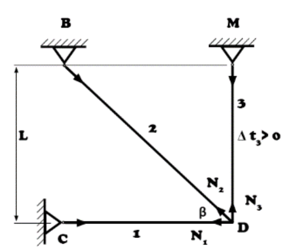
σ_1 and σ_2 are the two stresses. θ is the angle made by the oblique plane with the axis of the minor tensile stress. We don't need to know about any of the interpretations. We are just going to determine the lengths of the line segments in the diagram.

Suppose $\sigma_1 = 60\ \frac{N}{mm^2}$, $\sigma_2\ = 120\ \frac{N}{mm^2}$, $\theta = 30°$.

Determine the following lengths:

a) AC
b) BC
c) OB
d) CO
e) OE
f) OD
g) ED
h) DB
i) CE
j) and the value of ϕ

- A 3-rod truss is assembled. Then rod 3 is heated. This causes internal forces in the bars: N_1, N_2, and N_3.



Using Castigliano's Theorem, we find that the magnitudes of the forces are given by

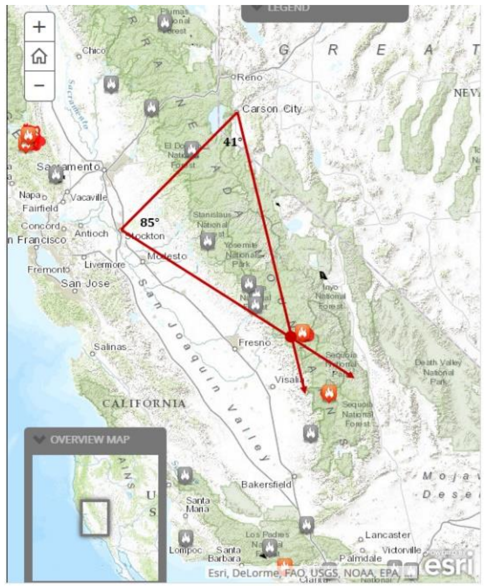
$$N_2 = \alpha \cdot \Delta t_3 \cdot E \cdot A \cdot \frac{\sin^2(\beta)}{1 + \cos^3(\beta) + \sin^3(\beta)}$$

$$N_1 = \alpha \cdot \Delta t_3 \cdot E \cdot A \cdot \frac{\sin^2(\beta) \cdot \cos(\beta)}{1 + \cos^3(\beta) + \sin^3(\beta)}$$

$$N_3 = \alpha \cdot \Delta t_3 \cdot E \cdot A \cdot \frac{\sin^3(\beta)}{1 + \cos^3(\beta) + \sin^3(\beta)}$$

a) Explain why N_1 would be $N_2 \cdot\ \cos(\beta)$
b) Explain why N_3 would be $N_2 \cdot\ \sin(\beta)$

- A forest fire is raging east of Fresno. Two ranger stations in Stockton, CA and Carson City, NV are triangulating the fire. Stockton and Carson City are 117 miles apart. From the line of sight between the two cities, they have measured the angles to the direction of the fire. Calculate the distance of the fire from each city.

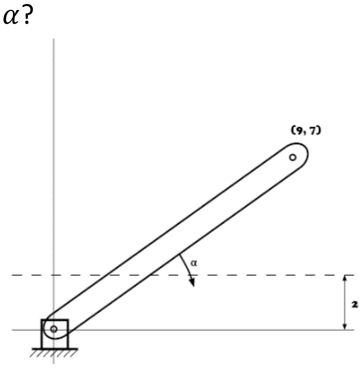


5c. apply inverse trigonometric functions. Students recognize if questions are asking for lengths or angle measurements and can phrase a solution plan. Using calculators, students can obtain angular measurements from linear measurements. *

Illustrations:

- A road sign quotes a 7% grade. Determine the angle that the road makes with the horizontal.

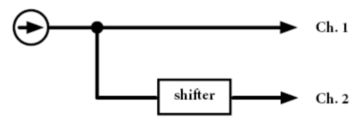
- A robot arm is currently positioned so that it reaches out to the point $(9\ in, 7\ in)$. The arm is to turn clockwise an angle α, so that it reaches out to a point a vertical distance of $2\ in$ above the pivot point. What is α?



5d. analyze and compare basic sine and cosine graphs. Students can graphically determine amplitude, period, phase shift, etc. *

Illustrations:

- A function generator is producing a sine wave at $5\ kHz$. The phase shifter is set for $0.02\ ms$. Draw the signals in channels 1 and 2.



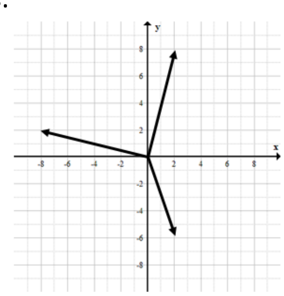
6. Vectors: Successful Technical Mathematics students are proficient with vectors. Students rephrase information in terms of vectors and easily moving between components and resultants.

The successful Technical Mathematics student can:

6a. represent vectors. Students represent vectors algebraically and graphically in either rectangular or polar coordinates. *

Illustrations:

- The arrows below describe three vectors. Express these vectors algebraically with rectangular and polar coordinates.



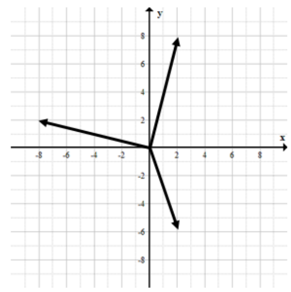
- Draw arrows representing $\langle -2, 5 \rangle$ and $\langle 3, 25° \rangle$.

- Determine the distance between the ends of the hands when the clock reads four o'clock, where the hour hand is 2.5 inches long and the minute hand is 3 inches long.

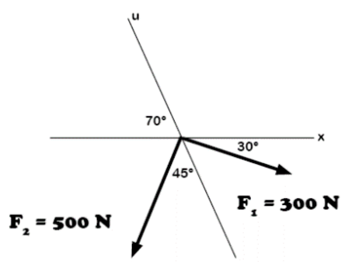
6b. perform arithmetic with vectors. Students represent vectors algebraically and graphically and perform arithmetic operations in either mode. *

Illustrations:

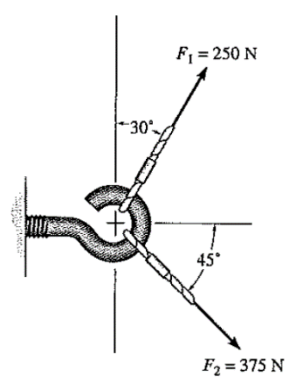
- Get the rectangular coordinates for the vectors below from the grid. Algebraically, add the three vectors together to get a resultant vector. Write the resultant vector in vector notation and then draw an arrow representing it.



- Here are two vectors, $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$. Determine their sum, $\overrightarrow{F_1} + \overrightarrow{F_2}$, and the angle this resultant vector makes with the positive x-axis.



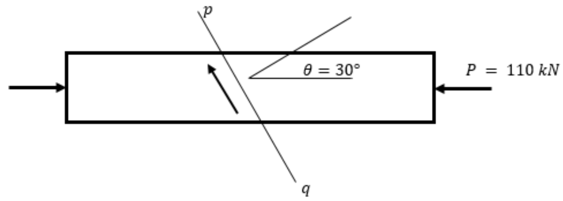
- Two ropes are pulling on an eye hook. Determine the magnitude and direction of the resultant force.



6c. decompose vectors. Students decompose measurements into components relative to given coordinate systems. *

Illustrations:

- A bar is compressed by an axial load $P = 110\ kN$. What is the vector component of P parallel to the $30°$ inclined section pq?



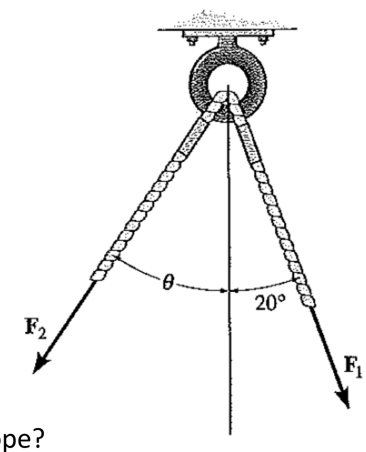
- Express $\langle 3, 9\rangle$ as a multiple of a unit vector.

- A ring fastened to a beam has two ropes attached. These two ropes exert forces on the ring.

  F_1 is applied at an angle of $20°$ from vertical.

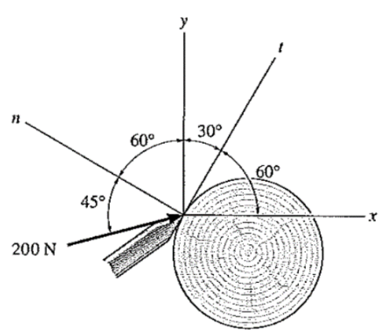
  F_2 is applied at an angle of $\theta°$ from vertical.

  It is desired that the combined force on the ring is $1000N$ straight down.



  If $\theta° = 30°$, then what are the forces, F_1 and F_2, in each rope?

- A wood dowel is spinning on a lathe. The chisel exerts a force of 200N at an angle illustrated in the diagram.



  a) What is the force in the x-direction?
  b) What is the force in the y-direction?
  c) What is the force in the t-direction?
  d) What is the force in the n-direction?

7. Complex Numbers: Successful Technical Mathematics students are comfortable with the arithmetic of complex numbers as well as their basic Geometry.

The successful Technical Mathematics student can:

7a. introduce arithmetic with complex numbers. Add, subtract, multiply, and divide. *

Illustrations:

- Compute $5 + 7i + (-3 + 2i)$; $4 + 2i + 6 - 3i$; and $(5 + 7i) \cdot (-3 + 2i)$

7b. connect algebraic expressions with points in the plane. $a + bi$ associated with (a, b). *

Illustrations:

- Plot dots in the Complex plane representing $5 + 7i$; $6 - 3i$; and $-1 - i$

7c. map complex arithmetic with vector arithmetic. *

Illustrations:

- Translate $5 + 7i - (-3 + 2i) = 8 + 5i$ into vector arithmetic
- Illustrate $5 + 7i - (-3 + 2i) = 8 + 5i$ graphically through arrows.

8. Functions: Successful Technical Mathematics students are proficient with dependencies. Students analyze the effect on dependent quantities from changes in independent quantities, whether this dependency is described algebraically or graphically.

The successful Technical Mathematics student can:

8a. communicate via function notation. *

Illustrations:

- The student uses $f(2)$ as a representation of a number
- The student can explain the difference between the quantities $f(x + h)$ and $f(x) + h$.

8b. evaluate functions via formulas or graphs. *

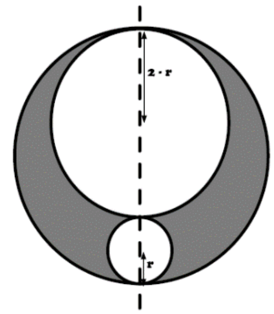
Illustrations:

- The student can evaluate a function using the graph.
- The student determines the domain and range of a function given its graph.
- The student can explain to a peer how to evaluate a given piecewise-defined function.

8c. obtain graphs via technology given formulas. *

Illustrations:

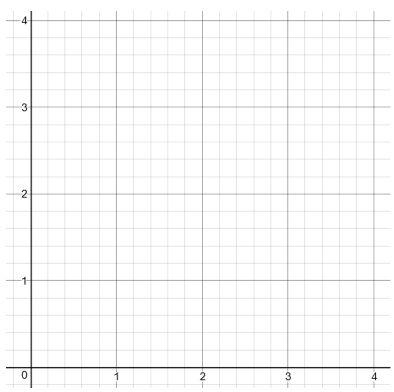
- A diameter runs through a circle of radius 2. Move out a distance r along the diameter and draw an internal tangent circle of radius r. And, also draw a tangent circle of radius $(2 - r)$ along the same diameter. See diagram.



  The outer circle has an area. When you remove the areas of the two inner circles, then you have the remaining area left over inside the outer circle. This leftover area changes if as you change r.

  If $r = 0$, then the leftover area is 0.
  If $r = 2$, then the leftover area is 0.
  If $0 < r < 2$, then the leftover area varies.

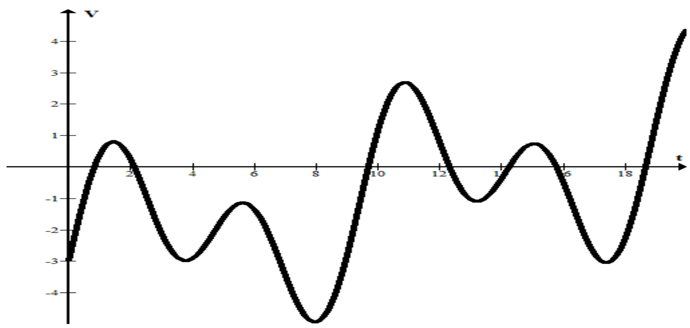
  a) Write a formula for this leftover area with respect to r.
  b) Go to desmos.com and graph your formula. Draw your graph here.



8d. analyze function behavior graphically. (increasing/decreasing, maximums/minimums) *

Illustrations:

- The graph of the voltage across a terminal in a circuit is graphed below. The voltage changes as the time, in seconds, changes.



a) What is the difference between a positive voltage and a positive change in voltage?
b) What is the difference between a negative voltage and a negative change in voltage?
c) Is it possible to have a positive voltage, but a negative change? Explain.
d) Is it possible to have a negative voltage, but a positive change? Explain.

8e. work explicitly with linear functions *

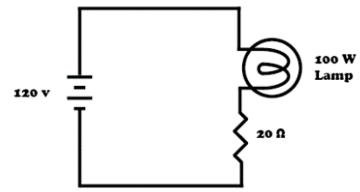
Illustrations:

- For the steel wire in 4f), α and E are constant, which means stress is a linear function of change in temperature.
  a) What is the formula?
  b) Sketch a graph of this linear function.
  c) What is the slope of your graph?
  d) What is the rate of change of stress with respect to change in temperature?
  e) What information does the rate of change give?

8f. work explicitly with quadratic functions. *

Illustrations:

- The circuit below has a voltage source supplying 120 volts to the circuit. There is a $100-watt$ lamp and a 20Ω resistor in series.



There is a voltage drop across the lamp, which we'll call V_L.

There is a voltage drop across the resistor, which we'll call V_R.

These two voltages drops have to add up to $120v$: $120 = V_L + V_R$

On the other hand, there is also a current flowing through the circuit, which we'll call, I. This current flows through the resistor, which gives us $V_R = 20\,I$.

The lamp consumes $100\ watts$ of power. Power equals volts times current : $100 = V_L\, I$. We can solve this for V_L to get $\frac{100}{I} = V_L$

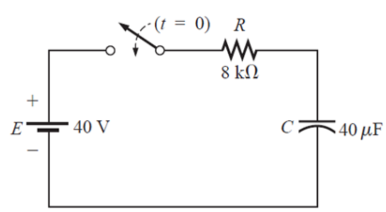
We now have both V_L and V_R written in terms of I and we can substitute these into the voltage equation, $120 = \frac{100}{I} + 20\,I$

Solve this equation for the possible currents.

8g. evaluate, graph, and graphically analyze exponential functions. *

Illustrations:

- The circuit shown below includes a $40V$ voltage source, an $8k\Omega$ resistor and a $40\mu F$ capacitor.



When the switch is closed, the current will begin flowing and the capacitor will begin to charge up. As the capacitor charges up, the voltage across the resistor drops. Eventually, the voltage across the capacitor is $40V$ and the voltage drop across the resistor is $40V$.

$$V_c(t) = 40V\left(1 - e^{-\frac{t}{R \cdot C}}\right)$$

$$V_R(t) = 40V\ e^{-\frac{t}{R \cdot C}}$$

Go to Desmos.com and graph both $V_c(t)$ and $V_R(t)$