Ohio | Department of Higher Education

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Ohio Transfer 36 Learning Outcomes

Announced- March 30, 2021

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Background

With the approved revisions of the Ohio Transfer 36 now in effect, current discipline groups will shift from course guidelines to now course learning outcomes. The shift from guidelines to learning outcomes will keep in line with efforts to clearly articulate what a student should know and be able to accomplish as a result of learning. Some areas have already shifted from guidelines to learning outcomes with the remainder of Ohio Transfer 36 discipline areas transitioning to this approach.

Arts and Humanities *(Updated March 30, 2021)*

The Ohio Transfer 36 requires at least 6 semester hours of course credit in Arts and Humanities.

- Students completing courses in the Arts and Humanities category should achieve the learning outcomes below through the study of humanities disciplines such as the arts, music, theatre, film, literature, philosophy, and history. Students must select courses from at least two different disciplines to fulfill Ohio Transfer 36 minimum requirements.
- Ohio Transfer 36 Arts and Humanities courses should be introductory-level courses that focus on the study of human endeavors spanning historical periods, regions, and/or cultures.
- Course materials should clearly articulate how students interact with primary sources, which may include (but are not limited to) works of art, music, theatre, film, literature, or philosophy.

Excluded courses:

- Remedial or developmental courses, special topics courses, narrowly focused courses, and technical or pre-technical courses.
- Courses that focus exclusively on content coverage without addressing the learning outcomes for the Ohio Transfer 36.
- Career preparation courses, non-credit continuing education courses, and life experience courses (unless life experience credit, such as military training or other prior learning experience, becomes approved in the future for an Ohio Transfer 36 credit by the statewide faculty review panel).
- Courses that are primarily designed for skill development (e.g., applied music lessons, studio art, symbolic or formal logic, theatre skills, creative writing, and foreign language). To be approved, foreign language courses must devote a majority of the course content to

literature and not be grammar and/or skills-based. Skills-based activities, whether graded or not, may support the learning process in an Ohio Transfer 36 Arts and Humanities course, as long as the primary focus or goal of the course is not skill development.

Learning Outcomes:

1. **Basic Knowledge:** employ principles, terminology, and methods from disciplines in the arts and humanities.

2. **Textual Analysis:** analyze, interpret, and/or evaluate primary works that are products of the human imagination and critical thought.

3. **Contextual Examination:** reflect on the creative process of products of the human imagination and critical thought.

4. **Breadth:** explain relationships among cultural and/or historical contexts.

5. **Communication:** convey concepts and evidence related to humanistic endeavors clearly and effectively.

English Composition *(Updated March 30, 2021)*

The Ohio Transfer 36 requires at least 3 semester hours of course credit in English Composition/Oral Communication. Use of the Ohio Transfer 36 Guidelines for English Composition was discontinued starting Fall 2012. All English Composition Ohio Transfer 36 courses approved under the guidelines were expired by Summer 2012 and replaced by First or Second Writing only when the course received an approval for either First or Second Writing.

- In order to be considered for First Writing and Second Writing Ohio Transfer 36 courses, each institutional course must meet all of the established learning outcomes. In addition, each set of learning outcomes has recommended credit hours, so that institutions will be able to design, match, and submit courses with a comparable and appropriate amount of credit to fulfill the learning outcomes.

- First Writing and Second Writing Ohio Transfer 36 courses must focus on the teaching, practice, and evaluation of expository writing and argumentative writing, although the course(s) may include other components. This focus must be reflected in statements of course learning outcomes and evaluation.

- Transfer students who have completed the Ohio Transfer 36 will not be subjected to a diagnostic placement test at the receiving institution unless one is also required of native students who have completed equivalent coursework.

Excluded courses:

- Remedial or developmental courses, special topics courses, narrowly focused courses, technical or pre-technical courses and skills-based courses.
- Courses that focus exclusively on content coverage without addressing the learning outcomes for the Ohio Transfer 36.
- Career preparation courses, non-credit continuing education courses, and life experience courses (unless life experience credit, such as military training or other prior learning experience, becomes approved in the future for an Ohio Transfer 36 credit by the statewide faculty review panel).

TME001: First Writing Course

Throughout the first course, students practice reading and writing in several genres.

Minimal Course Requirements

Students must compose a substantial amount and variety of work in order to demonstrate that they have met the learning outcomes for the first writing course.

Learning to write **and** ***writing to learn*** **are often discrete activities, but both should be part of the writing class. To that end, students in the first writing class will**

- Compose a variety of texts with opportunities to consider and clarify their ideas in light of response from others, including teachers and peers.
- Produce at least 5000 words of text that has been thoughtfully revised and copyedited to meet the expectations of particular rhetorical situations. Multimodal texts may be included as part of the overall body of work students produce in the course.
- Complete frequent low-stakes or writing-to-learn activities such as single-draft reading responses, journals, in-class efforts, and discovery drafts.

Learning Outcomes:

1. Rhetorical Knowledge

Students will develop their understanding of rhetorical situations as they read and write in several genres.

By the end of their first writing course, students should

- Understand how genre conventions shape the texts they read and should shape the texts they compose.
- Understand the possibilities of electronic media/technologies for composing and publishing texts for a variety of audiences.
- Compose texts that
    - Have a clear purpose.
    - Respond to the needs of intended audiences.
    - Assume an appropriate stance.
    - Adopt an appropriate voice, tone, style, and level of formality.
    - Use appropriate conventions of format and structure.
- Recognize common rhetorical strategies and appeals.
- As appropriate, attempt to employ rhetorical strategies and appeals in their own writing.

2. Critical Thinking, Reading, and Writing

Students will develop their critical thinking skills as they analyze model texts and secondary sources.

By the end of their first writing course, students should be able to

- Use reading and writing for inquiry, learning, thinking, and communicating.
- Locate and evaluate secondary research materials, including visual texts such as photographs, videos, or other materials.
- Analyze relationships among writer, text, and audience in linguistically diverse texts.
- Use various critical thinking strategies to analyze texts.
- Develop a clear line of reasoning and recognize how incorporating ideas and evidence from sources can strengthen their work.

3. Knowledge of Composing Processes

Students will work individually and collaboratively to hone their revising and editing skills.

By the end of their first writing course, students should be able to

- Recognize that writing is a flexible, recursive process that typically involves a series of activities, including generating ideas and text, drafting, revising, and editing.
- Use electronic environments to support writing tasks such as drafting, reviewing, revising, editing, and exploring texts.
- Discover and reconsider ideas through drafting, reviewing, and revising.
- Recognize the difference between revising and editing and understand why both processes are critical.
- Understand that writing is often collaborative and social. To demonstrate that understanding, students should be able to
    - Work with others to improve their own and others' texts.
    - Balance the advantages of relying on others with taking responsibility for their own work.
- Apply this understanding and recognition to make global and local revisions.

4. Knowledge of Conventions

Students will study genre conventions and apply appropriate conventions to their own work.

By the end of their first writing course, students should be able to

- Recognize the genre conventions for structure, paragraphing, tone, and mechanics employed in various rhetorical contexts.
- Use syntax, grammar, punctuation, and spelling appropriate to particular rhetorical situations.
- Select and employ appropriate conventions for structure, paragraphing, mechanics, format, and design.
- Acknowledge the work of others and use a standard documentation format as needed.

TME002: Second Writing Course

Throughout the second course, students critically read scholarly texts, learn about conventions for academic writing, and practice writing for various rhetorical situations. At some schools, students explore what "good writing" is as they learn about the different discourse conventions privileged by various academic disciplines; such courses are typically offered by the English department. At other schools, students explore what "good writing" is in one specific academic discipline; such courses may be taught by the English department, taught by another academic department, or team-taught by English faculty and faculty in another academic department. Regardless of the approach to the second course, the focus of the course must be *writing*, and students must be given opportunities to revise, reflect, and get personal feedback on their written work.

Minimal Course Requirements

Students must compose a substantial amount and variety of work in order to demonstrate that they have met the learning outcomes for the second course.

In the second writing course, students will again engage in both writing-to-learn and learning-to-write activities. To that end, they will

- Compose a variety of texts with opportunities for revision and response.
- Produce a minimum of 5000 total words of text that has been revised and copyedited for applicable rhetorical situations. Multimodal texts may be included as part of the overall body of work students produce in the course.
- Complete frequent low-stakes or writing-to-learn activities such as single-draft reading responses, journals, and in-class efforts, as well as discovery drafts.

Learning Outcomes:

1. **Rhetorical Knowledge**

   Students will further develop their understanding of rhetorical situations as they read academic texts and practice tailoring their work for specific audiences.

   The second writing course reinforces the rhetorical principles that students address in the first writing course. In addition, by the end of the second course, students should be able to

- Read academic texts and understand how disciplinary conventions shape the texts they read.
- Compose texts that respond to the needs of appropriate audiences, using suitable discourse conventions to shape those texts.
- Use academic conventions of format and structure when appropriate.

2. Critical Thinking, Reading, and Writing

Students will further develop their critical thinking skills as they analyze and synthesize academic texts.

The second writing course should reinforce the critical reading and thinking skills students developed in the first course. In addition, by the end of the second course, students should be able to

- Find and evaluate appropriate material from electronic and other sources.
- Locate, evaluate, organize, and use primary and secondary research material. Secondary research material should be collected from various sources, including journal articles and other scholarly texts found in library databases, other official databases (e.g., federal government databases), and informal electronic networks and internet sources.
- Analyze and critique sources in their writing.
- Juxtapose and integrate ideas and arguments from sources.
- Use strategies—such as interpretation, synthesis, response, critique, and design/redesign—to compose texts that integrate their original ideas with those from academic sources and other documents.

3. Knowledge of Composing Processes

Students will continue to hone their revision strategies and reflect critically on their writing practices.

The second class should reinforce the fact that writing is a flexible and recursive process. Because students often write more scholarly texts in the second course than they did in the first, practice in generating ideas and text, drafting, revising, and editing are even more important in the second class. By the end of the second class, students should be able to

- Select and apply appropriate writing processes to match the context.
- Revise for a variety of technologies and modalities.
- Use composition and revision as a means to discover and reconsider ideas.
- Reflect on the development of their revision strategies and consider how those strategies influence their work.

- Produce successive drafts of increasing quality.

4. Knowledge of Conventions

Students will study academic conventions and apply appropriate conventions to their own work.

The second writing course should reinforce and expand the knowledge of conventions students developed in the first writing course. In addition, by the end of the second writing course, students should be able to

- Understand why conventions vary.
- Recognize the genre conventions employed by various academic disciplines.
- Employ appropriate textual conventions for incorporating ideas from sources (e.g., introducing and incorporating quotations; quoting, paraphrasing, and summarizing).

Oral Communication *(Updated March 30, 2021)*

The Ohio Transfer 36 requires at least 3 semester hours of course credit in English Composition/Oral Communication.

- Courses in oral communication are an option for *elective* courses within the Ohio Transfer 36. If a student completes a course in oral communication but does not complete the entire Ohio Transfer 36, the course may only apply as an elective upon transfer because not all institutions have this requirement.
- Courses in oral communication are in addition to First and/or Second Writing and may not replace or substitute for composition courses.
- The major emphasis of the oral communication course must be extemporaneous public speaking (individual/group work) as reflected in statements of course learning outcomes and evaluation. The course(s) may include group presentations and argumentation. Typically, hybrid courses in oral communication include examination of communication theory but should concentrate on evaluated oral presentations as the primary focus of the course.
- Courses that include communication principles leading up to the study or understanding of the oral communication process in other forms (e.g., interviewing, interpersonal, dyads, listening) are not precluded if they meet the criterion for emphasis on extemporaneous speaking.

Excluded courses:

- Remedial or developmental courses, special topics courses, narrowly focused courses, technical or pre-technical courses and skills-based courses.
- Courses that focus exclusively on content coverage without addressing the learning outcomes for the Ohio Transfer 36.
- Career preparation courses, non-credit continuing education courses, and life experience courses (unless life experience credit, such as military training or other prior learning experience, becomes approved in the future for an Ohio Transfer 36 credit by the statewide faculty review panel).
- Courses in which the main focus is theory, the study of communication styles, or oral interpretation and performance (i.e., students should research and prepare their own oral presentations, not give a recitation of existing work).

TMOC: Oral Communication

Learning Outcomes:

1. **Preparation for performance/composition and organization of speeches**

Students develop speeches that are consistent and appropriate for the purpose, context, and audience.

Students should be able to present speeches that are consistent and appropriate for the purpose, context, and audience. In order to demonstrate this outcome, by the end of the public speaking course, students should be able to

- Apply organizational skills to construct speeches that are context appropriate. Give several informal and formal speeches in a variety of genres.
- Practice communicating for a variety of purposes and distinguish among those purposes.
- Recognize the importance of audience analysis and adaptation in public speaking. Develop and adapt messages, style, and delivery to meet the needs of diverse audiences. Use speeches to embrace difference, demonstrate diversity and inclusion, and understand relationships between cultures.
- Become fully informed about the subject matter by researching topics responsibly and ethically. Research may involve finding and retrieving information from personal experience as well as published sources.
- Critically examine sources for credibility, accuracy, relevance, and usefulness.

- Synthesize information from multiple sources to construct an argument. Appropriately cite sources.

2. Delivery of messages

Students present speeches using effective verbal and nonverbal delivery techniques and appropriate presentational aids.

By the end of their public speaking course, students should be able to

- Use appropriate and effective verbal and nonverbal delivery. Use delivery techniques (posture, gesture, eye contact, pauses, and vocal expressiveness) and language choices that make the speech understandable.
- Present well-developed and appropriately organized informative material and persuasive arguments.
- Use presentational aids or technology in ways that enhance speeches.
- Manage communication apprehension and increase confidence across communication contexts. Speakers should appear comfortable.
- Monitor and adjust the speech according to audience feedback. Manage time appropriately.
- Make clear distinctions between speakers' ideas and ideas of others.

3. Critical assessment of messages

Students critically and constructively evaluate their own and others' speeches.

By the end of their public speaking class, students should be able to critically and constructively evaluate their own speeches, as well as the speeches of classmates and professionals.

- When they are audience members, they should be attentive. Furthermore, they should use active-listening skills to objectively evaluate the speeches of others.
- When they are collaborating with peers, they should work to improve their own and others' speeches, balancing the advantages of relying on others with taking responsibility for their own work.

Natural Sciences *(Updated March 30, 2021)*

The Ohio Transfer 36 requires at least 6 semester hours of course credit in Natural Sciences, including at least one semester hour of course credit of Natural Sciences laboratory. Natural Sciences courses approved for inclusion within the Ohio Transfer 36 are introductory in nature, require college-level proficiencies appropriate to the course, and are taught at the lower division college level. Each course has a consistent content and a broad focus on one or more disciplines from within the physical and/or biological sciences, which include astronomy, biology, chemistry, environmental science, geology, physical geography and physics. Students completing courses in the Natural Science category should achieve the following learning outcomes through the study of natural sciences disciplines such as astronomy, biology, chemistry, environmental science, geology, physical geography, and physics.

As appropriate to the discipline, the course highlights the nature of science, the importance of experimental inquiry in the Natural Sciences, and the way in which such inquiry into the natural world leads scientists to formulate principles that provide universal explanations of diverse phenomena. The course fosters an understanding and appreciation that all applicable evidence must be integrated into scientific models of the universe, and that scientific models must evolve. A course that focuses primarily on content coverage, without addressing each of the Student Learning Outcomes described herein, is not suitable as an Ohio Transfer 36 Natural Sciences course.

In completing the Natural Sciences requirements within the Ohio Transfer 36, students will accurately understand and describe the scope of scientific study and core theories and practices, in either or both the physical and biological sciences, using appropriate discipline-related terminology.

Excluded courses:

- Remedial or developmental courses, special topics courses, narrowly focused courses, technical or pre-technical courses and skills-based courses.
- Courses that focus exclusively on content coverage without addressing the learning outcomes for the Ohio Transfer 36.
- Career preparation courses, non-credit continuing education courses, and life experience courses (unless life experience credit, such as military training or other prior learning experience, becomes approved in the future for an Ohio Transfer 36 credit by the statewide faculty review panel).

Learning Outcomes:

Upon completion of any approved Ohio Transfer 36 Natural Sciences course, students will be able to:

1. Understand the basic facts, principles, theories and methods of modern science.

2. Explain how scientific principles are formulated, evaluated, and either modified or validated.

3. Use current models and theories to describe, explain, or predict natural phenomena.

4. Apply scientific methods of inquiry appropriate to the discipline to gather data and draw evidence-based conclusions.

5. Demonstrate an understanding that scientific data must be reproducible but that it shows intrinsic variation and can have limitations.

6. Apply foundational knowledge and discipline-specific concepts to address issues or solve problems.

7. Explain how scientific principles are used in understanding the modern world, and understand the impact of science on the contemporary world.

8. Gather, comprehend, apply and communicate credible information on scientific topics, evaluate evidence-based scientific arguments in a logical fashion, and distinguish between scientific and non-scientific evidence and explanations.

Natural Sciences Laboratory Requirement: students will complete at least one course within the Natural Sciences Ohio Transfer 36 that includes a laboratory component. This laboratory component must carry at least one credit hour and involve at least 1,500 minutes of laboratory activities (an average of no less than two hours per week for a traditional 15-week semester). During the course, students will demonstrate the application of the methods and tools of scientific inquiry appropriate to the discipline, by actively and directly collecting, analyzing, and interpreting data, presenting findings, and using information to answer questions.

In addition to achieving the Student Learning Outcomes 1-8 detailed above, Ohio Transfer 36 approved courses that include a laboratory component[1] will achieve all the following student learning objectives in the equivalent of at least 10 weeks (~2/3) of the course's "laboratory activities":

- involves realistic measurements of physical quantities;
- involves data analysis, using data that are unique and/or physically authentic and that include random and/or systematic (natural) variability;
- includes realistic interactions with experimental apparatus, and realistic manipulation of tools/ instruments and/or observed objects in space and time;
- involves synchronous feedback[2] on safety (and consequences of unsafe actions), correctness of procedure, and progress toward experimental goals; and
- involves effective interaction with the instructor at several points during each lab activity.

Footnotes:

1. *Some disciplines, such as astronomy, meteorology, and ecology, are more amenable to achieving a quality virtual educational lab experience. By contrast, other disciplines, such as chemistry*, microbiology and physics, are much less likely to meet the expectations of an Ohio Transfer 36 science lab course if focused heavily on virtual lab experiences. [*The American Chemical Society has released a Position Statement on this issue: https://www.acs.org/content/acs/en/policy/publicpolicies/invest/computersimulations.html.]*
2. *Synchronous feedback on safety could be achieved using sophisticated computational approaches or by actual instructor feedback.*

Social and Behavioral Sciences *(Updated March 30, 2021)*

The Ohio Transfer 36 requires at least 6 semester hours of course credit in the Social and Behavioral Sciences. Ohio Transfer 36 Social and Behavioral Sciences courses should be introductory-level courses that explain the behavior of individuals and/or various groups in societies, economies, governments, and subcultures through empirical investigation and theoretical interpretation.

Excluded courses:

- Remedial or developmental courses, special topics courses, narrowly focused courses, technical or pre-technical courses and skills-based courses.
- Courses that focus exclusively on content coverage without addressing the learning outcomes for the Ohio Transfer 36.
- Career preparation courses, non-credit continuing education courses, life experience courses (unless life experience credit, such as military training or other prior learning experience, is approved in the future for an Ohio Transfer 36 credit by the statewide faculty review panel).

Students completing courses in the Social and Behavioral Sciences category should achieve the following learning outcomes through the study of social and behavioral sciences disciplines such as anthropology, economics, geography, history, political science, psychology and sociology. Students must select courses from at least two disciplines.

Learning Outcomes:

1. **Core Knowledge**: Students will be able to explain the primary terminology, concepts, and findings of the specific social and behavioral science discipline.

2. **Theory**: Students will be able explain the primary theoretical approaches used in the specific social and behavioral science discipline.

3. **Methodology**: Students will be able to explain the primary quantitative and qualitative research methods used in the specific social and behavioral science discipline.

4. **Values**: Students will be able to explain the primary ethical issues raised by the practice and findings of the specific social and behavioral science discipline.

5. **Evidence**: Students will be able to explain the range of relevant information sources in the specific social and behavioral science discipline

Mathematics, Statistics, and Logic *(Updated March 30, 2021)*

The Ohio Transfer 36 requires at least 3 semester hours of course credit in Mathematics, Statistics, and Logic, Ohio Transfer 36 Mathematics, Statistics, and Logic courses should be:

1. A credit-bearing, college-level course in Mathematics must use the standards required for high school graduation by the State of Ohio as a basis and must do at least one of the following: 1) broaden, or 2) deepen, or 3) extend the student's learning.
2. The course does not cover variable learning outcomes from term to term.
3. The course is not an upper-division course.
4. The course is in the areas of mathematics, or statistics, or logic.

Excluded courses:

- Remedial or developmental courses, special topics courses, narrowly focused courses, technical or pre-technical courses and skills-based courses.

- Courses that focus exclusively on content coverage without addressing the learning outcomes for the Ohio Transfer 36.

- Career preparation courses, non-credit continuing education courses, and life experience courses (unless life experience credit, such as military training or other prior learning experience, becomes approved in the future for an Ohio Transfer 36 credit by the statewide faculty review panel).

TMM001 - COLLEGE ALGEBRA *(Revised December 8, 2015; updated samples April 30, 2016)*

Typical Range: 3-4 Semester Hours

Recommendation: This course should significantly reflect the Mathematical Association of America's Committee on the Undergraduate Program in Mathematics (CUPM) subcommittee, Curriculum Renewal Across the First Two Years (CRAFTY), College Algebra Guidelines.

College Algebra provides students a college level academic experience that emphasizes the use of algebra and functions in problem solving and modeling, where solutions to problems in real-world situations are formulated, validated, and analyzed using mental, paper-and-pencil, algebraic and technology-based techniques as appropriate using a variety of mathematical notation. Students should develop a framework of problem-solving techniques (e.g., read the problem at least twice; define variables; sketch and label a diagram; list what is given; restate the question asked; identify variables and parameters; use analytical, numerical and graphical solution methods as appropriate; determine the plausibility of and interpret solutions). – Adapted from the MAA/CUPM CRAFTY 2007

College Algebra Guidelines To qualify for TMM001 (College Algebra), a course must achieve all of the following essential learning outcomes listed in this document (marked with an asterisk). The Sample Tasks are recommendations for types of activities that could be used in the course.

1. **Functions**: Successful College Algebra students demonstrate a deep understanding of functions whether they are described verbally, numerically, graphically, or algebraically (both explicitly and implicitly). Students should be proficient working with the following families of functions: linear, quadratic, higher-order polynomial, rational, exponential, logarithmic, radical, and piecewise-defined functions (including absolute value)

   The successful College Algebra student can:

   1a. Analyze functions. Routine analysis includes discussion of domain, range, zeros, general function behavior (increasing, decreasing, extrema, etc.). In addition to performing rote processes, the student can articulate reasons for choosing a particular process, recognize function families and anticipate behavior, and explain the implementation of a process (e.g., why certain real numbers are excluded from the domain of a given function).*

   Sample Tasks:

- The student determines the domain of a function described algebraically and gives reasons for domain restrictions.
- The student determines the range of a function given its graph.
- The student can explain to a peer how to evaluate a given piecewise-defined function.
- The student recognizes and accurately represents asymptotic behavior on the graph of an exponential function.
- The student can explain the difference between the quantities $f(x + h)$ and $f(x) + h$.
- The student can explain the difference between $f(x) + f(x)$ and $f(x) + f(y)$.

1b. Convert between different representations of a function.*

Sample Tasks:

- The student translates a function from a verbal description to an algebraic description to determine its domain.
- The student constructs a table or a graph to approximate the range of a function described algebraically.
- The student graphs a piecewise-defined function to determine intervals over which the function is increasing, decreasing, or constant.
- The student can formulate a possible equation for a function given a graph.
- The student can verbalize the function represented variously as: $y = x$ 2 , $h(t) = t$ 2 , and $\{(u, u$ 2) $: u\ is\ a\ real\ number\}$ as 'the squaring function.'
- The student uses knowledge of end behavior to adjust the viewing window of a graphing utility.
- The student can graph the equation $3u + 4t = 6$ both on the tu- and ut-axes and compare the slopes in each case.
- The student graphs $f(x) = x$ 4 + $2x$ 2 – 117, suspects symmetry about the y-axis, and proves the symmetry analytically by showing $f(-x) = f(x)$.
- Discuss domain and range of functions defined implicitly: y 3 + xy + x 117 = 0.

1c. Perform operations with functions including addition, subtraction, multiplication, division, composition, and inversion; connect properties of constituent functions to properties of the resultant function; and resolve a function into a sum, difference, product, quotient, and/or composite of functions.*

Sample Tasks:

- Given the formula for a rational function f, the student can simplify the difference quotient for f.
- Given the graph of f, the student constructs the graph of 1 f .
- The student can verbally describe the relationship between the graph of $y = f(x)$, and each of $y = f(|x|)$ and $y = |f(x)|$.

- Given an algebraic description for f and a graph for g, the student can determine values for the sum, difference, product, quotient, and composition of f and g.
- Given the graph of f and g, the student can determine the domain of $f\ g$ and $f \circ g$.
- Given formulas for $f(x)$ and $(f \circ g)(x)$, a student can create a formula for $g(x)$.
- The student can explain how to determine a formula for the composition of two piecewise-defined functions.
- Given the graph of a function, the student can determine if the function is invertible and, if so, graph the inverse.
- The student can find functions f, g, and h so that $F(t)$ = √ 3t−4 t+1 = $(f \circ (\ g\ h\ ))(t)$.
- Given the graph of a function f, the student can graph $y = 3f(1 - x) + 2$.
- Given the graphs of two functions, the student can determine if they appear to be related by a sequence of linear transformations.

2. Equations and Inequalities: Successful College Algebra students are proficient at solving a wide array of equations and inequalities involving linear, quadratic, higher-order polynomial, rational, exponential, logarithmic, radical, and piecewise-defined functions (including absolute value).

The successful College Algebra student can:

2a. Recognize function families as they appear in equations and inequalities and choose an appropriate solution methodology for a particular equation or inequality and can communicate reasons for that choice.*

Sample Tasks:

- The student can summarize a solution strategy for a given problem verbally, without actually solving the problem.
- The student can solve an equation by factoring and explain the connection to the zero product property of real (complex) numbers.
- The student can explain the steps taken to construct a sign diagram and use a sign diagram to solve an inequality.
- The student can solve an equation involving piecewise-defined functions.

2b. Use correct, consistent, and coherent notation throughout the solution process to a given equation or inequality.*

2c. Distinguish between exact and approximate solutions and which solution methodologies result in which kind of solutions.*

Sample Tasks:

- The student lists the exact values of the irrational zeros of a quadratic function and uses decimal approximations to sketch the graph.
- The student recognizes the need to approximate the solutions to $2 - x = e\ x$ and uses a graphing utility to do so.

2d. Demonstrate an understanding of the correspondence between the solution to an equation, the zero of a function, and the point of intersection of two curves. *

Sample Tasks:

- The student solves an equation algebraically and verifies the solution using a graphing utility.
- Given the graphs of two functions f and g, the student can approximate solutions to $f(x) = g(x)$. 2e. Solve for one variable in terms of another. *

Sample Tasks:

- The student can solve for y: $2y = x(y - 2)$.
- The student can write an equation for the volume of a box as a function of the height given relationships between the length, width, and height of the box.

2f. Solve systems of equations using substitution and/or elimination. *

3. Equivalencies: Successful College Algebra students are proficient in creating equivalencies in order to simplify expressions, solve equations and inequalities, or take advantage of a common structure or form.

The successful College Algebra student can:

3a. Purposefully create equivalences and indicate where they are valid. *

Sample Tasks:

- To graph $f(x) = (x\ 2{-}4)\ (x{-}2)$, the student simplifies to $f(x) = x + 2$, and graphs $y = x + 2$ with a hole at (2,4).
- To solve $log(x) + log(x - 2) = 1$, a student solves $log(x\ 2 - 2x) = 1$ and knows this procedure may result in extraneous solutions.
- A student solves $|2x - 3| + 3x = 2$ by rewriting the left hand side as a piecewisedefined function

3b. Recognize opportunities to create equivalencies in order to simplify workflow. *

Sample Tasks:

- A student recognizes y = 2 (x − 2) 3 + 1 as being related to y = x 3 via linear transformations and exploits this in order to sketch the graph.
- A student rewrites x 2 − 4x + 4y 2 = 0 in standard form in order to graph.

4. Modeling with Functions: Successful College Algebra students should have experience in using and creating mathematics which model a wide range of phenomena.

The successful College Algebra student can:

4a. Interpret the function correspondence and behavior of a given model in terms of the context of the model.*

Sample Tasks:

- Given a 'doomsday' model for population, the student can interpret the vertical asymptote as 'doomsday.'
- Given a model that models the temperature of a cup of coffee, the student can determine the horizontal asymptote of the graph of the model and interpret it as the limiting temperature of the coffee.
- Given a model which models the height of a model rocket off the ground, the student can find and interpret the zeros of the model.

4b. Create linear models from data and interpret slope as a rate of change.*

Sample Tasks:

- The student can create the conversion between Fahrenheit and Celsius.
- The student can create a piecewise linear function which models the cost of a mobile phone data plan.
- The student can recognize linear trends in data and create a piecewise linear function to model behavior.

4c. Determine parameters of a model given the form of the model and data.*

Sample Tasks:

- The student can find and interpret the decay constant given the half-life of a radioactive element.
- Given the graph of what appears to be a parabola, the student can use knowledge of the vertex and intercepts to create a formula for the function in the form $f(x)$ = $a(x - h)$ 2 + k, or $f(x)$ = ax 2 + bx + c, or $f(x)$ = $a(x - r1)(x - r2)$.

4d. Determine a reasonable applied domain for the model as well as articulate the limitations of the model.*

Sample Tasks:

- Given the model: $h(t)$ = 640t – 32 t 2 gives height in feet of a model rocket off the ground t seconds after liftoff, a student determines a reasonable applied domain for the model can be obtained by solving $h(t) > 0$.

5. Appropriate Use of Technology: Successful College Algebra students are proficient at choosing and applying technology to assist in analyzing functions.

The successful College Algebra student can:

5a. Anticipate the output from a graphing utility and make adjustments, as needed, in order to efficiently use the technology to solve a problem.*

Sample Tasks:

- A student uses end behavior and a table of values to determine a reasonable window within which to locate the solution to an optimization problem.
- A student can use algebra and technology to produce a detailed graph of $f(x)$ = 5(x 2−4) 1 2 (x 2+1) . 5b. Use technology to verify solutions to equations and inequalities obtained algebraically.*

5b. Use technology to verify solutions to equations and inequalities obtained algebraically. *

Sample Tasks:

- A student solves x 2 − 3x > 1 + x and checks the reasonableness of the solution graphically.

5c. Use technology to obtain solutions to equations to equations and inequalities which are difficult to obtain algebraically and know the difference between approximate and exact solutions. *

Sample Tasks:

- A student decides to solve e x = 1 − x by graphing Y1 = e x and Y2 = 1 − x and observes the solution appears to be $x = 0$. The student then verifies the solution algebraically.
- A student uses WolframAlpha to solve e x = 2 − x, and, having no idea what W(e 2) is, records the approximate answer of 0.44.

5d. Use technology and algebra in concert to locate and identify exact solutions. *

Sample Tasks:

- A student uses the Rational Zeros Theorem to help identify approximate zeros from a graphing utility, then uses the Factor Theorem to verify which real numbers are zeros.
- A student decides to solve $e\ x = 1 - x$ by graphing $Y1 = e\ x$ and $Y2 = 1 - x$ and observes the solution appears to be $x = 0$. The student then verifies the solution algebraically.

6. Reasons Mathematically: Successful college algebra students demonstrate a proficiency at reasoning mathematically.

The successful College Algebra student can:

6a. Recognize when a result (theorem) is applicable and use the result to make sound logical conclusions and provide counter-examples to conjectures. *

Sample Tasks:

- A student uses the factor theorem to partially factor a higher degree polynomial in order to help find the exact values of the remaining zeros.
- A student recognizes the equation $e\ 2x - e\ x = 5$ as quadratic in form and uses the quadratic formula to solve for $e\ x$.
- A student uses Descartes' Rule of Signs to prove $f(x) = x\ 3 + x - 3$ has no negative real zeros.
- A student realizes the vertex formula cannot be used to find the extreme values of a third degree polynomial.
- A student can find pairs of real numbers where $(a + b)\ 2$ and $a\ 2 + b\ 2$ are different.

TMM002 - PRECALCULUS *(Revised March 21, 2017)*

Typical Range: 5-6 Semester Hours

Recommendation: This course should significantly reflect the spirit of the Mathematical Association of America's Committee on the Undergraduate Program in Mathematics (CUPM), Curriculum Renewal Across the First Two Years (CRAFTY), College Algebra Guidelines.

College Algebra provides students a college-level academic experience that emphasizes the use of algebra and functions in problem solving and modeling, where solutions to problems in realworld situations are formulated, validated, and analyzed using mental, paper-and-pencil, algebraic and technology-based techniques as appropriate using a variety of mathematical notation. Students should develop a framework of problem-solving techniques (e.g., read the problem at least twice; define variables; sketch and label a diagram; list what is given; restate the question asked; identify variables and parameters; use analytical, numerical and graphical solution methods as appropriate; and determine the plausibility of and interpret solutions). – Adapted from the MAA/CUPM CRAFTY 2007, College Algebra Guidelines

To qualify for TMM002 (Precalculus), a course must achieve all of the following essential learning outcomes listed in this document (marked with an asterisk). These make up the bulk of a Precalculus course. Courses that contain only the essential learning outcomes are acceptable from the TMM002 review and approval standpoint. It is up to individual institutions to determine further adaptation of additional course learning outcomes of their choice to support their students' needs. The Sample Tasks are suggestions/ideas for types of activities that could be used in the course. The Sample Tasks are not requirements.

1. **Functions:** Successful Precalculus students demonstrate a deep understanding of functions whether they are described verbally, numerically, graphically, or algebraically (both explicitly and implicitly). Students should be proficient working with the following families of functions: linear, quadratic, higher-order polynomial, rational, exponential, logarithmic, radical, piecewise-defined (including absolute value), and periodic functions.

   The successful Precalculus student can:
   1a. Analyze functions. Routine analysis includes discussion of domain, range, zeros, general function behavior (increasing, decreasing, extrema, etc.), as well as periodic characteristics such as period, frequency, phase shift, and amplitude. In addition to performing rote processes, the student can articulate reasons for choosing a particular process, recognize function families and anticipate behavior, and explain the implementation of a process (e.g., why certain real numbers are excluded from the domain of a given function).*

   Sample Tasks:
   - The student determines the domain of a function described algebraically and gives reasons for domain restrictions.
   - The student determines the range of a function given its graph.
   - The student can explain to a peer how to evaluate a given piecewise-defined function.
   - The student recognizes and accurately represents asymptotic behavior on the graph of an exponential function.
   - The student can explain the difference between the quantities $f(x + h)$ and $f(x) + h$.
   - The student can explain the difference between $f(x) + f(x)$ and $f(x) + f(y)$.
   - The student can explain the inverse relationships for trigonometric functions, as well as explain domain and range restrictions and interpret geometrically.
   - Analyze compositions involving known functions: log(sin(t)) , e cos(x) , √cos(θ) , sin (1 φ) , e $-t$ sin(2u).
   - The student can explain why π 2 is excluded from the domain of the tangent function.
   - The student can explain the difference between $sin(2x)$ and 2 $sin(x)$.

1b. Convert between different representations of a function. *

Sample Tasks:

- The student translates a function from a verbal description to an algebraic description to determine its domain.
- The student constructs a table or a graph to approximate the range of a function described algebraically.
- The student graphs a piecewise-defined function to determine intervals over which the function is increasing, decreasing, or constant.
- The student can formulate a possible equation for a function given a graph.
- The student can verbalize the function represented variously as: $y = x^2$, $h(t) = t^2$, and $\{(u, u^2) : u \text{ is a real number}\}$ as 'the squaring function'; as: $y = \sin(x)$, $h(t) = \sin(t)$, and $\{(u, \sin(u)) : u \text{ is a real number}\}$ as 'the sine function.'
- The student uses knowledge of end or asymptotic behavior to adjust the viewing window of a graphing utility.
- The student can graph the equation $3u + 4t = 6$ both on the tu- and ut-axes and compare the slopes in each case.
- The student graphs $f(x) = x^4 + 2x^2 - 117$, suspects symmetry about the y-axis, and proves the symmetry analytically by showing $f(-x) = f(x)$.
- Discuss domain and range of functions defined implicitly: $y^3 + xy + x^{117} = 0$ or $\sin(x)$ $\cos(y) = 1/2$.

1c. Perform operations with functions including addition, subtraction, multiplication, division, composition, and inversion; connect properties of constituent functions to properties of the resultant function; and resolve a function into a sum, difference, product, quotient, and/or composite of functions.*

Sample Tasks:

- Given the formula for a rational function f, the student can simplify the difference quotient for f.
- Through extending the graphs of quotient functions from rational functions to include trigonometric functions, the student constructs the graph of $y = \sin(3x)$ $\cos(2x)$.
- Given the graph of f, the student constructs the graph of $1\ f$.
- The student can verbally describe the relationship between the graph of $y = f(x)$, and each of $y = f(|x|)$ and $y = |f(x)|$.
- Given an algebraic description for f and a graph for g, the student can determine values for the sum, difference, product, quotient, and composition of f and g.
- Given the graph of f and g, the student can determine the domain of $f\ g$ and $f \circ g$.
- Given formulas for $f(\theta)$ and $(f \circ g)(\theta)$, the student can create a formula for $g(\theta)$.

- The student can explain how to determine a formula for the composition of two piecewise-defined functions.
- Given the graph of a function, the student can determine if the function is invertible and, if so, graph the inverse or create formula.
- The student can find functions f, g, and h so that $F(t)$ = √ 3t−4 t+1 = (f ∘ (g h))(t) or $F(t)$ = √ 3 cos(t)−4 cos(t)+1 = (f ∘ (g h)) (t).
- Given the graph of a function f, the student can graph $y = 3f(1 - x) + 2$.
- Given the graphs of two functions, the student can determine if they appear to be related by a sequence of transformations.
- The student can interpret e $-y$ sin(3y) as a sinusoid with an exponentially decaying amplitude (envelope).

2. Geometry: Successful Precalculus students demonstrate a deep understanding of the measurements of right triangles, right triangles as building blocks of general triangles, and right triangles as a bridge between circular measurements and rectangular measurements.

The successful Precalculus student can:
2a. Analyze angles. Routine analysis of angle measurements, units, and arithmetic.*

Sample Tasks:
- The student can measure drawings of angles using degrees and radians and convert between the two systems.
- The student can estimate measurements of angles and sketch angles with given measurements.
- The student can extend absolute measurements to the plane adding a positive and negative direction.

2b. Analyze right triangles. Routine analysis of side lengths and angle measurements using trigonometric ratios/functions, as well as the Pythagorean Theorem.*

Sample Tasks:
- The student can solve right triangles numerically using trigonometric ratios and relationships.
- The student can compare similar triangles numerically.
- The student can describe relationships within or between right/similar triangles algebraically using trigonometric ratios and relationships.

2c. Analyze general triangles. Routine analysis of side lengths and angle measurements using trigonometric ratios/functions, as well as other relationships.*

Sample Tasks:

- The student can solve general triangles using trigonometric ratios and relationships including laws of sine and cosine.
- The student can compare similar triangles.
- The student can compute length and angle measurements inside complex drawings involving multiple geometric objects.
- The student can algebraically describe relationships inside complex drawings involving multiple geometric objects.

3. Equations and Inequalities: Successful Precalculus students are proficient at solving a wide array of equations and inequalities involving linear, quadratic, higher-order polynomial, rational, exponential, logarithmic, radical, piecewise-defined (including absolute value), and trigonometric functions.

The successful Precalculus student can:

3a. Recognize function families as they appear in equations and inequalities and choose an appropriate solution methodology for a particular equation or inequality, as well as communicate reasons for that choice.*

Sample Tasks:
- The student can summarize a solution strategy for a given problem verbally, without actually solving the problem.
- The student can solve an equation by factoring and explain the connection to the zero product property of real (complex) numbers.
- The student can explain the steps taken to construct a sign diagram and use a sign diagram to solve an inequality.
- The student can solve an equation involving piecewise-defined functions.
- The student can solve $2\ sin2\ (t) + 7\sin(t) - 4 = 0$ on a given interval.
- The student can solve $log4\ (\sin(t)) + log4\ (2\sin(t) + 7) = 1$ on a given interval.

3b. Use correct, consistent, and coherent notation throughout the solution process to a given equation or inequality.*

Sample Tasks:
- The student is comfortable with given function and variable names.
- The student can choose meaningful function and variable names given a situation to model.

3c. Distinguish between exact and approximate solutions and which solution methodologies result in which kind of solutions.*

Sample Tasks:

- The student lists the exact values of the irrational zeros of a quadratic function and uses decimal approximations to sketch the graph.
- The student recognizes the need to approximate the solutions to 2 − x = e x or sin(x) cos(y) = 1 2 and uses a graphing utility to do so. 3d. Demonstrate an understanding of the algebraic, functional, and geometric views of equation solutions. Solutions to equations can simultaneously serve multiple purposes by representing numbers satisfying an equation, zeros of a function, and intersection points of two curves.*

Sample Tasks:

- The student solves an equation algebraically and verifies the solution using a graphing utility.
- Given the graphs of two functions f and g, the student can approximate solutions to $f(x)$ = $g(x)$.

3e. Solve for one variable in terms of another.*

Sample Tasks:

- The student can solve for y: 2y = x(y − 2).
- The student can write an equation for the volume of a box as a function of the height given relationships between the length, width, and height of the box.

3f. Solve systems of equations using substitution and/or elimination.*

Sample Task:

- The student can solve 3x + 2y = 5 −x + 5y = 4 or x + y + x = 3 2x − y + 3z = 4 −x + 3y − z = 1.

3g. Cite domain restrictions resulting from solution decisions and situation restrictions and reflect these in solution set descriptions.*

Sample Tasks:

- The student can solve for θ: y = √cos(θ).
- The student can provide domain, range, and graph: y = √cos(θ) .

4. Equivalencies: Successful Precalculus students are proficient in creating equivalencies in order to simplify expressions, solve equations and inequalities, or take advantage of a common structure or form.

The successful Precalculus student can:

4a. Purposefully create equivalences and indicate where they are valid.*

Sample Tasks:

- To graph $f(x) = (x\ 2{-}4)\ (x{-}2)$, the student simplifies to $f(x) = x + 2$ and graphs $y = x + 2$ with a hole at (2,4).
- To solve $log(x) + log(x - 2) = 1$, the student solves $log(x\ 2 - 2x) = 1$ and knows this procedure may result in extraneous solutions.
- The student solves $|2x - 3| + 3x = 2$ by rewriting the left-hand side as a piecewise-defined function.
- To graph $f(t) = \tan(t)\ \cos(t)$, the student simplifies to $f(t) = \sin(t)$ and graphs $y = \sin(t)$ with holes at ($\pi\ 2 + \mathbb{Z}\pi$, ±1).
- To solve $\sqrt{\cos(4t)} = \sqrt{\sin(4t)}$, the student solves $\cos(4t) = \sin(4t)$ and knows this procedure may result in extraneous solutions.
- The student solves $|\cos(2\theta - 3)| + 3\ 2 = 2$ by rewriting the left-hand side as a piecewise-defined function.
- The student can rewrite formulas involving multiple occurrences of the variable to formulas involving a single occurrence. Write $a \sin(w\ t) + b \cos(w\ t)$ as $A \sin(w\ t + B)$ or $B \cos(w\ t + B)$.
- The student can rewrite sums as products to reveal attributes such as zeros, envelopes, and phase interference.

4b. Recognize opportunities to create equivalencies in order to simplify workflow.*

Sample Tasks:

- The student recognizes $y = 2\ (x - 2)\ 3 + 1$ as being related to $y = x\ 3$ via transformations and exploits this in order to sketch the graph.
- The student rewrites $x\ 2 - 4x + 4y\ 2 = 0$ in standard form in order to graph.
- The student recognizes $y = 2 \cos(\theta - 3) + 1$ as being related to $y = \cos(\theta)$ via transformations and exploits this in order to sketch the graph.
- The student simplifies given trigonometric formulas for graphing reasons.

4c. Become fluent with conversions using traditional equivalency families.* (e.g., $(\sin(t))\ 2 + (\cos(t))\ 2 = 1$; $(\tan(t))\ 2 + 1 = (\sec(t))\ 2$; sums/differences; products; double angle; Euler's Formula ($e\ i\theta = \cos(\theta) + i \sin(\theta)$); etc.)

Sample Tasks:

- The student can prove trigonometric identities.
- The student solves trigonometric equations

5. Modeling with Functions: Successful Precalculus students should have experience in using and creating mathematics which models a wide range of phenomena.

The successful Precalculus student can:

5a. Interpret the function correspondence and behavior of a given model in terms of the context of the model.*

Sample Tasks:

- Given a 'doomsday' model for population, the student can interpret the vertical asymptote as 'doomsday.'
- Given a model that represents the temperature of a cup of coffee, the student can determine the horizontal asymptote of the graph of the model and interpret it as the limiting temperature of the coffee.
- Given a model which represents the height of a model rocket off the ground, the student can find and interpret the zeros of the model.
- Given a sales model for lawn chairs, the student can interpret the periodicity, phase shift, and amplitude.
- Given a model of the daily temperature, the student can determine a periodic model and interpret it.
- Given a model of a playground swing's height off the ground, the student can explain the limiting height and where to find corresponding information in the formula.

5b. Create linear and periodic models from data and interpret slope as a rate of change.*

Sample Tasks:

- The student can create the conversion between Fahrenheit and Celsius.
- The student can create a piecewise linear function which models the cost of a mobile phone data plan.
- The student can recognize linear trends in data and create a piecewise linear function to model behavior.
- The student can create formulas for yearly measurements.
- The student can recognize periodic trends in data and create a function to model behavior.

5c. Determine parameters of a model given the form of the model and data.*

Sample Tasks:

- The student can find and interpret the decay constant given the half life of a radioactive element.

- Given the graph of what appears to be a parabola, the student can use knowledge of the vertex and intercepts to create a formula for the function in the form $f(x) = a(x - h)\ 2 + k$ or $f(x) = ax\ 2 + bx + c$ or $f(x) = a(x - r1)(x - r2)$.
- The student can describe effects of changing parameter values for amplitude, phase shift, etc.
- Given the graph of what appears to be a periodic function, the student can use knowledge of the intercepts, minimums/maximums, and asymptotes to create a formula for the function.

5d. Determine a reasonable applied domain for the model, as well as articulate the limitations of the model.*

Sample Tasks:

- Given the model $h(t) = 640t - 32\ t\ 2$ gives height in feet of a model rocket off the ground t seconds after liftoff, the student determines that a reasonable applied domain for the model can be obtained by solving $h(t) > 0$.
- Given the model $h(t) = 6 \tan(\ t\ 12)$ which gives height in feet of a model rocket off the ground t seconds after liftoff, the student determines a reasonable applied domain for the model.

6. Appropriate Use of Technology: Successful Precalculus students are proficient at choosing and applying technology to assist in analyzing functions.

The successful Precalculus student can:

6a. Anticipate the output from a graphing utility and make adjustments, as needed, in order to efficiently use the technology to solve a problem.*

Sample Tasks:

- The student uses end behavior and a table of values to determine a reasonable window within which to locate the solution to an optimization problem.
- The student can use algebra and technology to produce a detailed graph of $f(x) = 5(x\ 2{-}4)\ 1\ 2\ (x\ 2{+}1)$ or $f(\varphi) = \cos(\ \varphi\ 30) \sin(2\varphi - 1)$.

6b. Use technology to verify solutions to equations and inequalities obtained algebraically.*

Sample Task:

- The student solves $x\ 2 - 3x > 1 + x$ or $|\cos(2t - 3)| + 3\ 2 = 2$ and checks the reasonableness of the solution graphically.

6c. Use technology to obtain solutions to equations and inequalities which are difficult to obtain algebraically and know the difference between approximate and exact solutions.*

Sample Tasks:

- The student decides to solve $e^{x} = 1 - x$ by graphing $Y1 = e^{x}$ and $Y2 = 1 - x$ and observes the solution appears to be $x = 0$. The student then verifies the solution algebraically.
- The student decides to solve $e^{\sin(x)} = 1 - x$ by graphing $Y1 = e$^($\sin(x)$) and $Y2 = 1 - x$ and observes the solution appears to be $x = 0$. The student then verifies the solution algebraically.
- The student can approximate solutions to $e^{\sin(10x)} = 2 - x^{2}$.
- The student uses WolframAlpha to solve $e^{x} = 2 - x$ and, having no idea what $W(e^{2})$ is, records the approximate answer of 0.44.

6d. Use technology and algebra in concert to locate and identify exact solutions.*

Sample Tasks:

- The student uses the Rational Zeros Theorem to help identify approximate zeros from a graphing utility, then uses the Factor Theorem to verify which real numbers are zeros.
- The student decides to solve $e^{x} = 1 - x$ by graphing $Y1 = e^{x}$ and $Y2 = 1 - x$ and observes the solution appears to be $x = 0$. The student then verifies the solution algebraically.

7. Reasons Mathematically: Successful Precalculus students demonstrate a proficiency at reasoning mathematically.

The successful Precalculus student can:

7a. Recognize when a result (theorem) is applicable and use the result to make sound logical conclusions and to provide counter-examples to conjectures.*

Sample Tasks:

- The student uses the factor theorem to partially factor a higher degree polynomial in order to help find the exact values of the remaining zeros.
- The student recognizes the equation $e^{2x} - e^{x} = 5$ as quadratic in form and uses the quadratic formula to solve for e^{x}.
- The student uses Descartes' Rule of Signs to prove $f(x) = x^{3} + x - 3$ has no negative real zeros.
- The student realizes the vertex formula cannot be used to find the extreme values of a third degree polynomial.
- The student can find pairs of real numbers where $(a + b)^{2}$ and $a^{2} + b^{2}$ are different.

- The student recognizes the equation $(\cos(x))$ 2 – $\sin(x)$ = 5 as quadratic in form and uses the quadratic formula or factoring to solve for $\sin(x)$. Then the student can determine values of x.
- After identifying forms, the student uses methods from previous courses.
- The student applies characteristics of functions to reason that equations cannot have a solution.
- The student rephrases an equation as information about a function and then identifies solutions based on function analysis.

Additional Optional Learning Outcomes
The Ohio Transfer 36 Mathematics, Statistics, and Logic Statewide Faculty Review Panel stresses that the essential learning outcomes marked with an asterisk make up the bulk of a Precalculus course and needs to continue as the focus of this course. Courses that contain only the essential learning outcomes are acceptable from the TMM002 review and approval standpoint. It is up to individual institutions to determine further adaptation of additional course learning outcomes of their choice to support their students' needs. The Statewide Review Panel will not use the additional optional learning outcomes for evaluative purposes and emphasizes that institutions must consider them as optional.
Some institutions expressed an interest in including additional optional learning outcomes to gain such guidance as possibilities to support students' needs. The Ohio Transfer 36 Mathematics, Statistics, and Logic Statewide Faculty Review Panel developed a few examples in this section, but please know that the additional learning outcomes are absolutely optional and not required. If your institution chooses to explore additional learning outcome(s), the examples below are simply suggestions and not restricted or exhaustive of possibilities. Please note that the samples provided here were crafted as a way to show how to purposefully tie back to the associated required items and to deem as extensions of the essential learning outcomes.

2. Geometry: Successful Precalculus students demonstrate a deep understanding of the measurements of right triangles, right triangles as building blocks of general triangles, and right triangles as a bridge between circular measurements and rectangular measurements.

The successful Precalculus student can:

2d. Describe two-dimensional position using rectangular and polar coordinates, vectors, and parametric equations; demonstrate fluency between any two of these systems; and recognize when one representation would be useful over another in simplifying workflow.

Sample Tasks:
- The student can translate between rectangular and polar coordinates.
- The student can draw curves described by equations involving rectangular and polar coordinates.

- The student can discuss geometric characteristics of curves defined implicitly: sin(x) cos(y) = 1 2 (level curves).
- The student can discuss geometric characteristics of curves defined parametrically.
- The student can discuss the difference between a curve and a parameterization.
- The student can discuss the difference between a function and a curve.
- The student successfully finds all points of intersection on the graphs of r = 4 cos(2 θ) and r = 2 by understanding that any given point has multiple representations.
- The student can translate curve descriptions into the language of vectors.
- The student can discuss direction vectors and lines described parametrically.

2e. Interpret the result of vector computations geometrically and within the confines of a particular applied context (e.g., forces).

Sample Tasks:

- The student can define vectors, their arithmetic, their representation, and interpretations.
- The student can decompose vectors into normal and parallel components.
- The student can interpret the result of a vector computation as a change in location in the plane or as the net force acting on an object. 2f. Represent conic sections algebraically via equations of two variables and graphically by drawing curves. Sample Tasks:
- The student can perform the process "completing the square" transforming the equation into a standard form.
- The student can draw curves representing conic sections.
- The student can solve systems of equations involving linear and quadratic functions.
- The student can parametrize conic curves.

8. Sequences and Series: Successful Precalculus students are proficient in manipulating sequences and series, as well as approximating functions with series.

The successful Precalculus student can:

8a. Represent sequences verbally, graphically, and algebraically, including both the general term and recursively.

Sample Tasks:

- The student can graph {an } = {(1 2) n }, including asymptotic behavior.
- The student defines {bn } by b0 = 1; bn+1 = bn 2 and determines a closed-form formula for the general term.

8b. Write series in summation notation and represent as sequences of partial sums verbally, numerically, and graphically.

Sample Tasks:

- The student defines $\{an \}$ by $an = \sum k\ k2+1\ n\ k=0$ and determines N such that $aN > 2$.
- The student defines $\{bn \}$ by $bn = \sum 1\ k(k+1)\ n\ k=0$ and determines a closed-form formula for the general term. In addition, the student can determine bn as $n \to \infty$.
- The student can apply algebraic properties of sequences and series.

8c. Identify and express the general term of arithmetic and geometric sequences and write the sum of arithmetic and geometric series.

Sample Tasks:

- The student defines $\{an \}$ by $an = 3 + 2n$ and determines $a13$.
- The student defines $\{bn \}$ by $bn = \sum 3 + 2k\ n\ k=0$ and determines a closed-form formula for the general term. In addition, the student can evaluate $b13$.

8d. Use “limits” of geometric series as functions by converting between series representation and closed forms, as well as using this bridge for composition and recentering.

Sample Tasks:

- The student defines $\{bn \}$ by $bn = \sum (\ 1\ 2\ )\ k\ n\ k=0$ and determines a closed-form formula for the general term. In addition, the student can evaluate the series.
- The student defines $f(n)$ by $f(n) = \sum (\ 1\ 2\ )\ k\ n\ k=0$ and determines a closed-form formula for $f(n)$.
- The student defines $f(x) = 1\ 1-x = \sum x\ \infty\ k\ k=0$ and determines a series representation for $1\ 3-2x$. In addition, the student can determine a series representation for $1\ 3-(\ 5-x\ 2\ )$ and determine a closed-form formula for $f(x) = \sum (x - 1)\ \infty\ k\ k=0$.

TMM003 - TRIGONOMETRY *(Revised March 21, 2017)*

Typical Range: 3-4 Semester Hours

Recommendation: This course should significantly reflect the spirit of the Mathematical Association of America's Committee on the Undergraduate Program in Mathematics (CUPM), Curriculum Renewal Across the First Two Years (CRAFTY), College Algebra Guidelines.

College Algebra provides students a college-level academic experience that emphasizes the use of algebra and functions in problem solving and modeling, where solutions to problems in real-world situations are formulated, validated, and analyzed using mental, paper-and-pencil, algebraic and technology-based techniques as appropriate using a variety of mathematical notation. Students should develop a framework of problem-solving techniques (e.g., read the problem at least twice; define variables; sketch and label a diagram; list what is given; restate the question asked; identify variables and parameters; use analytical, numerical and graphical solution methods as appropriate; and determine the plausibility of and interpret solutions). – Adapted from the MAA/CUPM CRAFTY 2007, College Algebra Guidelines

To qualify for TMM003 (Trigonometry), a course must achieve all of the following essential learning outcomes listed in this document (marked with an asterisk). These make up the bulk of a Trigonometry course. Courses that contain only the essential learning outcomes are acceptable from the TMM003 review and approval standpoint. It is up to individual institutions to determine further adaptation of additional course learning outcomes of their choice to support their students' needs. The Sample Tasks are suggestions/ideas for types of activities that could be used in the course. The Sample Tasks are not requirements.

1. Functions: Successful Trigonometry students demonstrate a deep understanding of periodic functions. This includes trigonometric functions whether they are described verbally, numerically, graphically, pictorially, geometrically, or algebraically.

The successful Trigonometry student can:

1a. Analyze functions. Routine analysis includes discussion of domain, range, zeros, and general function behavior (increasing, decreasing, extrema, etc.), as well as periodic characteristics such as period, frequency, phase shift, and amplitude with emphasis on functions derived from the geometry of the unit circle. In addition to performing rote processes, the student can articulate reasons for choosing a particular process, recognize function families and anticipate behavior, and explain the implementation of a process (e.g., why certain real numbers are excluded from the domain or range of a given function).*

Sample Tasks:

- The student determines the domain and range of a function described algebraically and gives reasons for restrictions.
- The student determines the domain and range of a function given its graph.
- The student can explain to a peer how to evaluate a given piecewise-defined function.
- The student recognizes and accurately represents asymptotic behavior on the graph of a trigonometric function.
- The student can explain the difference between the quantities $f(x + h)$ and $f(x) + h$.
- The student can explain the difference between $f(x) + f(x)$ and $f(x) + f(y)$.
- The student can explain the inverse relationships for trigonometric functions, as well as explain domain and range restrictions and interpret geometrically.

- Analyze compositions involving known functions: log(sin(t)) , e cos(x) , √cos(θ) , sin (1 φ) , e −t sin(2u).
- The student can explain why π 2 is excluded from the domain of the tangent function.
- The student can explain the difference between $sin(2x)$ and 2 $sin(x)$.

1b. Convert between different representations of a function.*

Sample Tasks:

- The student translates a function from a verbal description to an algebraic description to determine its domain.
- The student constructs a table or a graph to approximate the range of a function described algebraically.
- The student graphs a piecewise-defined function to determine intervals over which the function is increasing, decreasing, or constant.
- The student can formulate a possible equation for a function given a graph.
- The student can verbalize the function represented variously as: y = sin(x), $h(t)$ = sin(t), and {(u, sin(u)) : u $is\ a\ real\ number$} as ‘the sine function.’
- The student uses knowledge of end or asymptotic behavior to adjust the viewing window of a graphing utility.
- The student suspects symmetry about an axis, the origin, or another line by analyzing a graph and proves the symmetry analytically.
- The student can discuss domain and range of functions defined implicitly: sin(x) cos(y) = 1/2 .

1c. Perform operations with functions including addition, subtraction, multiplication, division, composition, and inversion; connect properties of constituent functions to properties of the resultant function; and resolve a function into a sum, difference, product, quotient, and/or composite of functions.*

Sample Tasks:

- Through extending the graphs of quotient functions from rational functions to include trigonometric functions, the student constructs the graph of y = sin(3x) cos(2x) .
- The student can verbally describe the relationship between the graph of $y = f(t)$ and each of $y = f(|t|)$ and $y = |f(t)|$.
- Given an algebraic description for f and a graph for g, the student can determine values for the sum, difference, product, quotient, and composition of f and g.
- Given the graph of f and g, the student can determine the domain of f g and $f \circ g$.
- Given formulas for $f(\theta)$ and $(f \circ g)(\theta)$, the student can create a formula for $g(\theta)$.
- The student can explain how to determine a formula for the composition of two piecewise-defined functions.

- Given the graph/formula of a function, the student can determine if the function is invertible and, if so, graph the inverse or create formula.
- The student can find functions f, g, and h so that $F(t)$ = √ 3 cos(t) − 4 cos(t) + 1 = (f ∘ (g h)) (t).
- Given the graph of a function f, the student can graph $y = 3f(1 - x) + 2$.
- Given the graphs of two functions, the student can determine if they appear to be related by a sequence of linear transformations.
- The student can interpret e $-y$ $sin(3y)$ as a sinusoid with an exponentially decaying amplitude (envelope).

2. Geometry: Successful Trigonometry students demonstrate a deep understanding of the measurements of right triangles, right triangles as building blocks of general triangles, and right triangles as a bridge between circular measurements and rectangular measurements.

The successful Trigonometry student can:
2a. Analyze angles. Routine analysis of angle measurements, units, and arithmetic.*

Sample Tasks:
- The student can measure drawings of angles using degrees and radians and convert between the two systems.
- The student can estimate measurements of angles and sketch angles with given measurements.
- The student can extend absolute measurements to the plane adding a positive and negative direction.

2b. Analyze right triangles. Routine analysis of side lengths and angle measurements using trigonometric ratios/functions, as well as the Pythagorean Theorem.*

Sample Tasks:
- The student can solve right triangles numerically using trigonometric ratios and relationships.
- The student can compare similar triangles numerically.
- The student can describe relationships within or between right/similar triangles algebraically using trigonometric ratios and relationships.

2c. Analyze general triangles. Routine analysis of side lengths and angle measurements using trigonometric ratios/functions, as well as other relationships.*

Sample Tasks:
- The student can solve general triangles using trigonometric ratios and relationships including laws of sine and cosine.

- The student can compare similar triangles.
- The student can compute length and angle measurements inside complex drawings involving multiple geometric objects.
- The student can algebraically describe relationships inside complex drawings involving multiple geometric objects. 3. Equations and Inequalities: Successful Trigonometry students are proficient at solving a wide array of equations and inequalities involving trigonometric functions. The successful Trigonometry student can:

3a. Recognize function construction/algebra as it appears in equations and inequalities and choose an appropriate solution methodology for a particular equation or inequality, as well as communicate reasons for that choice.*

Sample Tasks:

- The student can summarize a solution strategy for a given problem verbally, without actually solving the problem.
- The student can solve an equation by factoring and explain the connection to the zero product property of real (complex) numbers.
- The student can explain the steps taken to construct a sign diagram and use a sign diagram to solve an inequality.
- The student can solve an equation involving piecewise-defined functions.
- The student can solve 2 sin2 (t) + 7 sin(t) − 4 = 0 on a given interval.
- The student can solve log4 (sin(t)) + log4 (2 sin(t) + 7) = 1 on a given interval.

3b. Use correct, consistent, and coherent notation throughout the solution process to a given equation or inequality.*

Sample Tasks:

- The student is comfortable with given function and variable names.
- The student can choose meaningful function and variable names given a situation to model.

3c. Distinguish between exact and approximate solutions and which solution methodologies result in which kind of solutions.*

Sample Tasks:

- The student lists the exact values of the irrational zeros of a quadratic function and uses decimal approximations to sketch the graph.
- The student recognizes the need to approximate the solutions to sin(x) cos(y) = 1 2 and uses a graphing utility to do so.

3d. Demonstrate an understanding of the algebraic, functional, and geometric views of equation solutions. Solutions to equations can simultaneously serve multiple purposes by representing numbers satisfying an equation, zeros of a function, and intersection points of two curves.*

Sample Tasks:

- The student solves an equation algebraically and verifies the solution using a graphing utility.
- Given the graphs of two functions f and g, the student can approximate solutions to $f(x) = g(x)$.

3e. Cite domain restrictions resulting from solution decisions and situation restrictions and reflect these in solution set descriptions.*

Sample Tasks:

- The student can solve for θ: $y = \sqrt{\cos(\theta)}$.
- The student can provide domain, range, and graph: $y = \sqrt{\cos(\theta)}$.

4. Equivalencies: Successful Trigonometry students are proficient in creating equivalencies in order to simplify expressions, solve equations and inequalities, or take advantage of a common structure or form.

The successful Trigonometry student can:

4a. Purposefully create equivalences and indicate where they are valid.*

Sample Tasks:

- To graph $f(t) = \tan(t)\cos(t)$, the student simplifies to $f(t) = \sin(t)$ and graphs $y = \sin(t)$ with holes at $(\pi\ 2 + \mathbb{Z}\pi, \pm 1)$.
- To solve $\sqrt{\cos(4t)} = \sqrt{\sin(4t)}$, the student solves $\cos(4t) = \sin(4t)$ and knows this procedure may result in extraneous solutions.
- The student solves $|\cos(2\theta - 3)| + 3\ 2 = 2$ by rewriting the left-hand side as a piecewise-defined function.
- The student can rewrite formulas involving multiple occurrences of the variable to formulas involving a single occurrence. Write $a \sin(w\,t) + b \cos(w\,t)$ as $A \sin(w\,t + B)$ or $B \cos(w\,t + B)$.
- The student can rewrite sums as products to reveal attributes such as zeros, envelopes, and phase interference.

4b. Recognize opportunities to create equivalencies in order to simplify workflow.*

Sample Tasks:

- The student recognizes $y = 2\cos(\theta - 3) + 1$ as being related to $y = \cos(\theta)$ via linear transformations and exploits this in order to sketch the graph.
- The student simplifies given trigonometric formulas for graphing reasons.

4c. Become Fluent with conversions using traditional equivalency families.* [e.g., (sin(t)) 2 + (cos(t)) 2 = 1 ; (tan(t)) 2 + 1 = (sec(t)) 2 ; sums/differences; products; double angle, Euler's Formula ($e\ i\theta$ = cos(θ) + i sin(θ)), etc.]

Sample Tasks:

- The student can prove trigonometric identities.
- The student solves trigonometric equations.

5. Modeling with Functions: Successful Trigonometry students should have experience in using and creating mathematics which models a wide range of phenomena.
The successful Trigonometry student can:
5a. Interpret the function correspondence and behavior of a given model in terms of the context of the model.*

Sample Tasks:

- Given a sales model for lawn chairs, the student can interpret the periodicity, phase shift, and amplitude.
- Given a model of the daily temperature, the student can determine a periodic model and interpret it.
- Given a model of a playground swing's height off the ground, the student can explain the limiting height and where to find corresponding information in the formula.

5b. Create periodic models from data.*

Sample Tasks:

- The student can create formulas for yearly measurements.
- The student can recognize periodic trends in data and create a function to model behavior.

5c. Determine parameters of a model given the form of the model and data.*

Sample Tasks:

- The student can describe effects of changing parameter values for amplitude, phase shift, etc.
- Given the graph of what appears to be a periodic function, the student can use knowledge of the intercepts, minimums/maximums, and asymptotes to create a formula for the function.

5d. Determine a reasonable applied domain for the model, as well as articulate the limitations of the model.*

Sample Task:

- Given the model $h(t)$ = 6 tan (t 12) which gives height in feet of a model rocket off the ground t seconds after liftoff, the student determines a reasonable applied domain for the model.

6. Appropriate Use of Technology: Successful Trigonometry students are proficient at choosing and applying technology to assist in analyzing functions.

The successful Trigonometry student can:

6a. Anticipate the output from a graphing utility and make adjustments, as needed, in order to efficiently use the technology to solve a problem.*
Sample Tasks:

- The student uses end behavior and a table of values to determine a reasonable window within which to locate the solution to an optimization problem.
- The student can use algebra and technology to produce a detailed graph of $f(\varphi)$ = cos (φ 30) sin(2φ − 1).

6b. Use technology to verify solutions to equations and inequalities obtained algebraically.*

Sample Task:

- The student solves |cos(2t − 3)| + 3 2 = 2 and checks the reasonableness of the solution graphically.

6c. Use technology to obtain solutions to equations and inequalities which are difficult to obtain algebraically and know the difference between approximate and exact solutions.*

Sample Tasks:

- The student decides to solve e sin(x) = 1 − x by graphing $Y1$ = e^(sin(x)) and $Y2$ = 1 − x and observes the solution appears to be x = 0. The student then verifies the solution algebraically.
- The student can approximate solutions to e sin(10x) = 2 − x 2 .

7. Reasons Mathematically: Successful Trigonometry students demonstrate a proficiency at reasoning mathematically.

The successful Trigonometry student can:

7a. Recognize when a result (theorem) is applicable and use the result to make sound logical conclusions and to provide counter-examples to conjectures. *

Sample Tasks:

- The student recognizes the equation (cos(x)) 2 – sin(x) = 5 as quadratic in form and uses the quadratic formula or factoring to solve for sin(x). Then the student can determine values of x.
- After identifying forms, the student uses methods from previous courses.
- The student applies characteristics of functions to reason that equations cannot have a solution.
- The student rephrases an equation as information about a function and then identifies solutions based on function analysis.

TMM005 – Calculus I *(Updated August 23, 2018)*

Traditionally, a Calculus I course has been described by the content it presents to students. However, recommendations from projects including Common Vision1 as well as reports such as the Committee on the Undergraduate Program in Mathematics2 tell us that content makes only half a successful course. A successful Calculus I course must also give equal consideration to student engagement. This revision of the TMM005 guidelines follows the spirit of these recommendations. Although the content of Calculus I remains essentially as it was, this content is rephrased through an active learning lens.

The guidelines listed below do not alter the material content of Calculus I in any significant manner. Calculus I still includes a numerical, graphical, and algebraic investigation of limits; functional interpretations of limits; continuity; derivative definition, rules, and theorems; graphical interpretations of the derivative as well as rates of change between variables; higher-order derivatives; curve sketching; function analysis; optimization; and an introduction to integral calculus including antiderivatives, areas of planar regions, substitution, and the Fundamental Theorem of Calculus. All of this content is again included and described in the guidelines below. The goal of this revision is to phrase these in terms of student engagement and student outcomes.

While the material content of Calculus I remains steady, we hope our Calculus I courses continue to evolve. As the Common Vision report cites “the status quo is unacceptable.” By intentionally rephrasing the guidelines, we hope to spark ideas for the other half of the course – student engagement. To that end, there are an overwhelming number of sample tasks phrased from a student perspective. These are not included as requirements. There is no implication of coverage. There is no suggested weighting. They are not directed at the content half of the course. They reference the engagement half of the course. They are included as a first step towards elevating our thinking of active learning, student engagement, and student outcomes in Calculus I. Placement of sample tasks is not a stipulation nor a restriction. The sample tasks are simply encouragement to faculty to elevate the importance of student engagement and suggest a level of learning. Ultimately, TMM005 courses should expose our students to the same material and also collectively engage our students in this material as much as possible. The nouns of calculus have long been established. We now need to promote the verbs.

Successful Calculus students demonstrate a deep understanding of functions whether they are described verbally, numerically, graphically, or algebraically (both explicitly and implicitly). Students proficiently work in detail with the following families of functions: linear, quadratic, higher-order polynomial, rational, exponential, logarithmic, radical, trigonometric, piecewise-defined functions, and combinations, compositions, and inverses of these. Students are adept with the tools of differentiation and integration and their application toward situational goals. Students articulate the relationships underlying rate-of-growth and accumulation. Finally, successful Calculus students offer observations, suggestions, and conclusions to an investigative discussion as well as respond to remarks by others.

In a Calculus I (TMM005) course, students should:

- develop effective thinking and communication skills;
- operate at a high level of detail;
- state problems carefully, articulate assumptions, understand the importance of precise definition, and reason logically to conclusions;
- identify and model essential features of a complex situation, modify models as necessary for tractability, and draw useful conclusions;
- deduce general principles from particular instances;
- use and compare analytical, visual, and numerical perspectives in exploring mathematics;
- assess the correctness of solutions, create and explore examples, carry out mathematical experiments, and devise and test conjectures;
- recognize and make mathematically rigorous arguments;
- read mathematics with understanding;
- communicate mathematical ideas clearly and coherently both verbally and in writing to audiences of varying mathematical sophistication;
- approach mathematical problems with curiosity and creativity, persist in the face of difficulties, and work creatively and self-sufficiently with mathematics;
- learn to link applications and theory;
- learn to use technological tools; and
- develop mathematical independence and experience open-ended inquiry.
  – Adapted from the MAA/CUPM 2015 Curriculum Guide

To qualify for TMM005 (Calculus I), a course must achieve all of the following essential learning outcomes listed in this document (marked with an asterisk). These make up the bulk of a Calculus I course. Courses that contain only the essential learning outcomes are acceptable from the TMM005 review and approval standpoint. It is up to individual institutions to determine further adaptation of additional course learning outcomes of their choice to support their students' needs. In addition, individual institutions will determine their own level of student engagement and manner of implementation. These guidelines simply seek to foster thinking in this direction.

1. Limits and Continuity: Successful Calculus students demonstrate a deep understanding of the concepts of limit and continuity whether described verbally, numerically, graphically, or algebraically (both explicitly and implicitly). At the foundational level, students examine images and preimages of open intervals while comparing properties as the intervals contract. Illustrating and explaining this mapping is at the heart of understanding limits. At the procedural level, students identify common hurdles within the limiting process and select appropriate algebraic options. At the functional level, students categorize limit properties, recognize functions involved in limiting situations, and strategically invoke the broader principles of continuity when establishing consequences.

The successful Calculus student can:
1a. analyze limits graphically and numerically. The student visually describes the limiting process with graphs of functions. The student produces/reads tables of function values which illustrate limit properties.*

Sample Tasks:

- The student concludes nested intervals are converging to a number or not converging to a number.
- The student visually explains how nested domain intervals correspond to nested range interval and guess limiting values.
- Given a graph of the piecewise defined function $G(t)$ = { $3t + 5$ $t < 2$ $2t + 1$ $2 \leq t$ the student explains why the image of every open interval around 2 in the domain contains function values greater than 5.
- Given the graph of a function, a range number, r, and an open interval around r on the vertical axis, the student draws an open interval on the horizontal axis that maps into the range interval.
- The student describes numbers that are getting closer to d, but greater than (or less than) d, and where they are positioned on the horizontal axis. The student maps these to the vertical axis.
- The student reads/writes limit notation ($\lim_{x \to 5} f(x) = 8$) and connects the notational pieces to the graph of a function.
- The student produces graphs for which $H(5) = 7$, yet $\lim_{k \to 5} H(k) \neq 7$, $\lim_{k \to 5^+} H(k) \neq 7$, and/or $\lim_{k \to 5^-} H(k) \neq 7$.
- The student produces graphs for which $H(5) \neq 7$, yet $\lim_{k \to 5} H(k) = 7$.
- The student uses technology to extend the examination to functions defined implicitly via equations. For example, student graphs a curve defined via an equation, chooses a

section of curve that defines a function, hypothesizes limiting values based on graph, and then uses this information to help with an algebraic examination.

- The student uses a graphing utility1 to compare limiting behavior of different families of functions. For example, compare limiting behavior of polynomial and exponential functions. For instance, lim$x$→∞ x 1000000000000000000000000 e x .
- The student discusses “near” in a limiting context.
- The student illustrates limiting process with tables of function values.
- The student illustrates failure of limiting process with tables of function values.
- The student illustrates the difference between two-sided and one-sided limits with tables of function values.
- The student uses a computer algebra system4 to produce tables of values with accuracies unavailable via calculators or by hand. For example, lim$x$→∞ x 10000 e x = 0. Provide a lower limit on x such that x 10000 e x < 0.5.
- The student uses a computer algebra system4 to create domain intervals that map into cited range intervals. For example, if lim t→8.7 (t − 6.5) 5(t + 8) 2(t − 9) = L, then find d such that the domain interval (8.7 − d, 8.7 + d) maps into the range interval (L − 0.01, L + 0.01).

1b. find limits algebraically. The student organizes a well-formed presentation of the details involved in the limiting process via formulas.*

Sample Tasks:

- The student explicitly applies limit rules using correct notation throughout.
- The student explains a plan of algebraic manipulation for evaluating a limit, before executing the plan.
- The student describes algebraic hurdles that might arise for particular types of formulas.
- The student categorizes types of formulas with types of techniques.
- The student explains the value of techniques (e.g., how does a conjugate help?).
- The student analyzes limits of piecewise-defined functions.
- The student explains how indeterminate forms can have a limiting value.
- The student algebraically evaluates limits of the form lim h→0 $f(a$+$h)$−$f(h)$ h .
- The student states which limit rules they are using as they proceed through a computation.
- The student identifies the assumptions of limit rules and can provide counterexamples that justify the need for those assumptions.

1c. recognize and explain limits at infinity. The student furthers his/her understanding of infinity and how to logically work with unboundedness.*

Sample Tasks:

- The student explains the "existence" of M such that lim $t\rightarrow\infty$ $g(t)$ = 7 describes behavior of $g(t)$ for the interval (M, ∞).
- The student classifies functions in order of dominance for limits at infinity.
- The student carries out algebraic limits at infinity.
- The student compares asymptotic behavior to simpler elentary functions.

1d. communicate fluently about the concept of continuity.*

Sample Tasks:

- The student explicitly applies the limit definition of continuity.
- The student constructs logical arguments for discontinuities.
- The student creates functions where different aspects of the definition fail.
- The student develops an argument about the continuity at 0 for the following functions and verbally explains the differences: s0 (θ) = { sin (1 θ) $\theta \neq$ 0 0 θ = 0 s1 (θ) = { θ sin(1 θ) $\theta \neq$ 0 0 θ = 0
- The student defends the assertion that R1 (x) = 1 x is continuous on its implied domain while R2 (x) = { 1 x $x \neq$ 0 0 x = 0 is not.
- The student discusses possible images of connected and disconnected pieces of the domain.
- The student classifies different types of discontinuities.
- The student hypothesizes on the continuity of the composition of functions.
- The student draws graphs of functions which demonstrate the need for each requirement in the Intermediate Value Theorem.
- The student uses a graphing utility3 to produce discontinuous functions whose graphs look continuous.
- The student points out limitations to technology. For example, $s(\theta)$ = 5 + 0.00000001 sin(θ).

2. Differentiation: Successful Calculus students demonstrate an extensive understanding of the concept of differentiation from the details of specific procedures to the logical reasoning of abstracting relationships. Students are comfortable with the algebraic details presented by the definition of the derivative and differentiation rules. Students manipulate algebraic representations to reveal and explain properties and characteristics of derivatives, explicitly and implicitly. Students include properties and characteristics of the derivative into their analysis of situational models. Students articulate the necessity of derivative conditions in abstract reasoning.

The successful Calculus student can:

2a. apply the definition of the derivative to differentiate a function at a number and extend to an interval, choose appropriate differentiation rules and apply them, and parse formulas for application of the chain rule.*

Sample Tasks:

- The student evaluates lim $h \to 0$ $f(a+h)-f(a)$ h for the number 'a' in the domain.
- The student interacts with $D(x)$ = lim $h \to 0$ $f(x+h)-f(x)$ h as a formula for a function.
- The student summarizes possible problems with the derivative definition and corresponding graphical characteristics.
- The student relates the differences between continuity and differentiation.
- The student paraphrases the proof of rules of differentiation.
- The student devises formulas for functions whose derivative matches a given formula.
- The student applies rules in succession obtaining derivatives of higher order.
- The student uses a graphing utility3 to differentiate between the derivative at a number vs. over an interval. Graph $f'(x)$ and $g(x)$ = $f(x+h)-f(x)$ h over an interval for various values of h. Discuss the size of h to bring the approximation to within a tolerance over the interval. Discuss intervals where the approximation is better and worse.
- The student uses a graphing utility3 to illustrate the difficulty of differentiation at domain numbers associated with corners in graphs or discontinuities in piecewise-defined functions. For example, graph the derivative of the absolute value function.

2b. operate the derivative as a tool. The derivative measures rates of change, and the student is able to utilize this tool within the framework of a functional model, including connecting the slope of a tangent line with the value of the derivative.*

Sample Tasks:

- The student builds an equation of the tangent line using derivative value.
- The student speculates about the change in function values relative to the change in domain values based on values of the derivative.
- The student explains meaning of rate of change in context.
- The student relates average rate of change to instantaneous rate of change.
- The student formulates a description for a linear approximating function.
- The student approximates function values.
- The student describes increasing and decreasing over an interval.
- The student compares increasing/decreasing over a domain interval to increasing/decreasing at a number in the domain.
- The student expresses the importance of open intervals and interval investigation in general.
- The student partitions the domain into pieces where the function is increasing, decreasing, or constant.

- The student phrases rate of change in terms of independent and dependent variables.
- The student compares changes in functions within a parameterized scenario.
- The student builds a function model for measured characteristics within a situation.
- The student translates situational restrictions or requirements into aspects of the modelling function.
- The student transfers function analysis back to applications and arrives at contextual conclusions and consequences.

2c. work with implicitly defined functions. The student begins to expand his/her idea of function beyond a representation where the dependent variable is isolated on one side of an equation.*

Sample Tasks:

- The student partitions a curve into pieces that defined a functional relation.
- The student discovers characteristics of a curve that dictate elements of partition.
- The student cites domain and range restrictions for implicit function definition.
- The student uses an equation description of a curve to evaluate function values.
- The student uses an equation description of a curve to evaluate derivative values.
- The student uses a parameterization of a curve to evaluate function values.
- The student uses a parameterization of a curve to evaluate derivative values.
- The student can switch orientation by switching the roles of the independent and dependent variables.
- The student constructs linear approximations.
- The student deduces the rule for the derivative of an inverse function from dy $dx \cdot dx$ dy = 1.
- The student describes situations by relating rates of change and analyzes such situations.
- The student explains relationships between the growth rates of dependent and independent measurements.
- The student discusses relationships between the growth rates of parameterized quantities.
- The student uses a graphing utility3 to visually suggest domain and range restrictions in creating implicitly defined functions. For example, graph the Siluroid Folium, (x 2 + y 2) 2 − 4 x n (x 2 − y 2) = 0 - identify sections of the curve that define y as a function of x and approximate values of dy dx for values of x. Identify points where a domain interval cannot be created where y is defined as a function of x. Switch roles: view y as the independent variable and x the dependent variable.
- The student uses a graphing utility3 to investigate the relation between curves and tangent lines.

- The student uses a graphing utility3 to illustrate difficulties related to tangent lines at particular points on a curve and relates the curve characteristics of such points back to the derivative. What is the equation of the tangent line at (0, 0) on $y = |x|$?

3. Graphing and Optimization: Successful Calculus students fully analyze situations described by functions. Students develop their own strategies and tactics and explain how their plan will coalesce into applicable information. Students summarize their plans, including statements of the situational goal; recognize the types of functions involved; appropriately apply derivatives, limits, and function properties; offer reasoning for their choices and decisions when information was sought; and purposefully arrive at conclusions.

The successful Calculus student can:

3a. identify critical numbers and extrema values. The student separates function characteristics from features of the graph.*

Sample Tasks:

- The student states an explicit definition of a global maximum/minimum.
- The student states an explicit definition of a local maximum/minimum.
- The student applies a definition to justify maximum/minimum values of a piecewise-defined function. Justify that $H(y)$ = { $3y + 2$ $y \leq 4$ $-y + 3$ $y > 4$ has a maximum at 4, but $R(y)$ = { $3y + 2$ $y < 4$ $-y + 3$ $y \geq 4$ does not.
- The student identifies endpoints of intervals and places of discontinuity as places where extrema occur or fail.
- The student explains how the derivative is used to locate critical numbers.
- The student produces a graph of a function with given characteristics.
- The student explains the ideas behind the first and second derivative tests.
- The student applies the first and second derivative test.
- The student uses a graphing utility3 to suggest places to look for extrema.
- The student uses a graphing utility3 to illustrate the derivative tests. For example, graph the derivative of $k(t)$ and identify/classify extrema candidates for $k(t)$. For instance, graph the derivative of $R(y)$ = { $3y + 2$ $y < 4$ $3y - 3$ $y \geq 4$

3b. sketch curves/graphs of functions using derivatives and limits. The student acquires a level of competency with visual representations. This is crucial to using technology meaningfully.*

Sample Tasks:

- The student organizes his/her own plan to analyze a function including: o intervals where a function is increasing/decreasing, o concavity, o global/local maximums/minimums, o asymptotes, o discontinuities, and o intercepts.

- The student presents a coherent report on his/her plan for analysis.
- The student draws appropriate conclusions and visually represents them.
- The student convinces other students of the validity of his/her plan and subsequent graph.
- The student uses technology to suggest points of focus for analysis.
- The student uses technology for comparison and contrast to his/her reasoning.
- The student identifies the limits of technology and explains how to decode the correct information from the display. For example, fluctuations too small for the display of the graphing calculator. Or, fluctuations that coincide with the pixel dimensions and thus hide the fluctuations.

3c. optimize quantities in applied problems. The student develops some fluency with the application of calculus tools to physical situations modeled by functions.*

Sample Tasks:

- The student organizes his/her own plan to optimize a function.
- The student presents a coherent report of his/her conclusions.
- The student judges conclusions.

4. Integration: Successful Calculus students can reverse the differentiation process. Working symbolically, students recognize the algebraic result of the chain rule, parse expressions into pieces based on their compositional position, and formulate a reasonable antiderivative. Students can quickly apply the chain rule to their suggested antiderivative, identify differences, and effectively alter their antiderivative. As tools, students use definite integrals to describe the accumulation of changes and the antiderivative to measure accumulation. Students communicate this relationship between rates of change and accumulation via the Fundamental Theorem of Calculus.

The successful Calculus student can:

4a. create antiderivatives. The student is fluent with the differentiation – antidifferentiation relationship and uses the FTC and integration by substitution.*

Sample Tasks:

- Given a derivative formula, the student maps out which part was produced by the chain rule.
- Given a derivative, the student creates a possible original function.
- The student can articulate his/her strategies for identifying chain rule outcomes within integrand.
- The student generalizes characteristics to families of antiderivatives.

- The student creates his/her own auxiliary functions with respect to the chain rule and, via substitution, rewrites the integral to better adhere to chain rule structure.
- The student converts integration limits according to substitution function.
- The student recovers format of original function after successful integration via substitution.
- The student investigates "+C" on separate connected components of the domain. For example, the antiderivative of $f(x)$ = 1 x is $F(x)$ = { ln($-x$) + C x < 0 ln(x) + D x > 0

4b. measure area of bounded planar regions. Given a planar region whose boundary curves are described by equations, the student describes the situation in terms of functions and accompanying integration setup.*

Sample Tasks:

- The student symbolizes area measurement with a definite integral.
- The student approximates area measurement via Riemann sums.
- The student evaluates limit of Riemann sums for value of definite integral and area.
- The student measures area with a definite integral.
- The student uses a computer algebra system4 to calculate Riemann sums and hypothesize. For example, identify a value for n such that Riemann sum is within a given tolerance of area value. Compare Riemann sums for different choices of evaluation number inside subintervals.

4c. understand the Fundamental Theorem of Calculus (FTC). The FTC connects the measurements of rates of change and accumulation. Understanding this relationship is intrinsic to an understanding of calculus.*

Sample Tasks:

- The student paraphrases the FTC and its purpose.
- The student calculates area measurement via definite integral and FTC.
- The student operates $F(x)$ = ∫ $f(t)$ dt x a as a function of x.
- The student hypothesizes about the derivative of $F(x)$ = ∫ $f(t)$ dt x a .
- The student uses a computer algebra system4 to investigate and hypothesize about the FTC.
- The student uses technology to explain the importance of the intervals in the FTC.
- The student uses technology to explain the importance of continuity in the FTC.
- The student uses technology to explain the meaning of differentiability in the FTC.
- The student uses technology to hypothesize on these connections. For example, graph the derivative of $F(x)$ = ∫ 3t 2 − 2t + 5 dt x 2 and $f(x)$ = 3x 2 − 2x + 5. Or, graph $F(x+h)-F(x)$ h and $f(x)$ = 3x 2 − 2x + 5 for various values of h.

TMM006 – Calculus II *(Updated August 23, 2018)*

Traditionally, a Calculus II course has been described by the content it presents to students. However, recommendations from projects including Common Vision1 as well as reports such as the Committee on the Undergraduate Program in Mathematics2 tell us that content makes only half a successful course. A successful Calculus II course must also give equal consideration to student engagement. This revision of the TMM006 guidelines follows the spirit of these recommendations. Although the content of Calculus II remains essentially as it was, this content is rephrased through an active learning lens.

The guidelines listed below do not alter the material content of Calculus II in any significant manner. Calculus II still includes using antiderivatives to evaluate definite integrals; a variety of applications to model physical, biological, or economic situations; geometric applications emphasizing Riemann sums; integration techniques such as substitution, by parts, trigonometric substitution, and partial fractions; evaluating indeterminate forms, including L'Hôpital's Rule; improper integrals; sequence and series convergence, including comparison, ratio, root, integral, and alternating tests; Taylor polynomials; power series representation, differentiation, and integration; and curve analysis whether described parametrically or in polar form along with area measurement of regions bounded by such curves. All of this content is again included and described in the guidelines below. The goal of this revision is to phrase these in terms of student engagement and student outcomes.

While the material content of Calculus II remains steady, we hope our Calculus II courses continue to evolve. As the Common Vision report cites "the status quo is unacceptable." By intentionally rephrasing the guidelines, we hope to spark ideas for the other half of the course – student engagement. To that end, there are an overwhelming number of sample tasks phrased from a student perspective. These are not included as requirements. There is no implication of coverage. There is no suggested weighting. They are not directed at the content half of the course. They reference the engagement half of the course. They are included as a first step towards elevating our thinking of active learning, student engagement, and student outcomes in Calculus II. Placement of sample tasks is not a stipulation nor a restriction. The sample tasks are simply encouragement to faculty to elevate the importance of student engagement and suggest a level of learning. Ultimately, TMM006 courses should expose our students to the same material and also collectively engage our students in this material as much as possible. The nouns of calculus have long been established. We now need to promote the verbs.

Successful Calculus students demonstrate a deep understanding of functions whether they are described verbally, numerically, graphically, or algebraically (both explicitly and implicitly). Students proficiently work in detail with the following families of functions: linear, quadratic, higher-order polynomial, rational, exponential, logarithmic, radical, trigonometric, piecewise-defined functions, and combinations, compositions, and inverses of these. Students are adept with the tools of differentiation and integration and their application toward situational goals. Students articulate the relationships underlying rate-of-growth and accumulation. Finally, successful Calculus students offer observations, suggestions, and conclusions to an investigative discussion as well as respond to remarks by others.

In a Calculus II (TMM006) course, students should:

- develop effective thinking and communication skills;
- operate at a high level of detail;
- state problems carefully, articulate assumptions, understand the importance of precise definition, and reason logically to conclusions;
- identify and model essential features of a complex situation, modify models as necessary for tractability, and draw useful conclusions;
- deduce general principles from particular instances;
- use and compare analytical, visual, and numerical perspectives in exploring mathematics;
- assess the correctness of solutions, create and explore examples, carry out mathematical experiments, and devise and test conjectures;
- recognize and make mathematically rigorous arguments;
- read mathematics with understanding;
- communicate mathematical ideas clearly and coherently both verbally and in writing to audiences of varying mathematical sophistication;
- approach mathematical problems with curiosity and creativity, persist in the face of difficulties, and work creatively and self-sufficiently with mathematics;
- learn to link applications and theory;
- learn to use technological tools; and
- develop mathematical independence and experience open-ended inquiry.
  – Adapted from the MAA/CUPM 2015 Curriculum Guide

To qualify for TMM006 (Calculus II), a course must achieve all of the following essential learning outcomes listed in this document (marked with an asterisk). These make up the bulk of a Calculus II course. Courses that contain only the essential learning outcomes are acceptable from the TMM006 review and approval standpoint. It is up to individual institutions to determine further adaptation of additional course learning outcomes of their choice to support their students' needs. In addition, individual institutions will determine their own level of student engagement and manner of implementation. These guidelines simply seek to foster thinking in this direction.

1. Applications of Definite Integral: Successful Calculus students model aspects of situations with functions. When the measurements under investigation are best described in terms of accumulation, then calculus students distinguish the varying characteristic, identify relationships, and build integral functions that measure the desired situational aspect. Calculus students describe and explain their integration models. Calculus students participate in discovery activities where they criticize and defend ideas.

The successful Calculus student can:

1a. model geometric measurements such as area, volume of solids of revolution, arc length, area of surfaces of revolution, and centroids with integration tools, including setting up an approximating Riemann sum and representing its limit as a definite integral.*

Sample Tasks:

- The student interprets and describes accumulation with Riemann sums.
- The student constructs definite integrals as limits of Riemann sums.
- The student models areas of 2-dimensional regions bounded by curves described via Cartesian equations, polar equations, or within parameterized coordinate systems.
- The student models volumes of 3-dimensional regions bounded by surfaces described via Cartesian equations, polar equations, or within parameterized coordinate systems.
- The student calculates area and volumes by applying antiderivative rules.
- The student explains connections between integrand expressions and given situations involving arc length or area of surface.
- The student designs integrals for length or surface area measurements.
- The student visualizes and predicts location of centroids.
- The student measures centroid coordinates via definite integrals.
- The student criticizes choice of coordinates or integration variable.

1b. model measurements within situations related to STEM fields.*

Sample Tasks:

- The student describes a measurement of work as a Riemann sum.
- The student describes a measurement of work as definite integral.
- The student adapts Riemann sums to accumulation situations.
- The student formulates models with respect to situations involving fluid forces in terms of Riemann sums.
- The student verbalizes Riemann sum structure within physical, biological, and economic situations.

1c. approximate accumulation measurements. [This guideline is not required, but many institutions include this in various ways as a foundational competency within their course.]

Sample Tasks:

- The student formulates definite integrals as a limit of Riemann sums.
- The student illustrates how definite integrals are approximated with finite sums.
- The student discusses error size.
- The student demonstrates the application of the Trapezoidal Rule.
- The student leads a group of students in the application of Simpson's Rule.

- The student participates in a group comparing Runge-Kutta algorithms using a computer.

2. Integration Techniques: Successful Calculus students demonstrate an extensive understanding of the concept of integration from the details of specific procedures to the logical reasoning of abstracting relationships. Students are comfortable with the algebraic manipulation used to rewrite integrands and integration limits. Students can articulate their choices and decisions when applying integration techniques.

The successful Calculus student can:

2a. use antiderivatives to evaluate definite integrals and employ a variety of integration techniques, including substitution, integration by parts, trigonometric substitution, and partial fraction decomposition.*

Sample Tasks:

- The student relates the benefits of chosen substitutions.
- The student criticizes substitution choices and predicts their ineffectiveness.
- The student identifies algebraic hurdles within integrands.
- The student speculates on types of techniques appropriate to algebraic hurdles.
- The student forecasts a list of steps to follow and expected outcomes.
- The student chooses appropriate integration techniques and applies them logically.
- The student classifies types of integrands according to techniques.
- The student illustrates application of integration by parts.
- The student demonstrates application of trigonometric substitution.
- The student explains application of partial fraction decomposition.

3. Improper Limits: Successful Calculus students are comfortable dealing with infinity. Students have an intuition about how functions compare in their end-behavior or near singularities. Students can precisely phrase situations involving infinity.

The successful Calculus student can:

3a. identify indeterminate forms within limits. The student reasons on his/her own that an indeterminate form is present. The student chooses an appropriate action. The student executes his/her plan and explains the whole process.*

Sample Tasks:

- The student analyzes function for possible singularities.
- The student classifies types of indeterminate forms.

- The student recognizes types of indeterminate forms.
- The student hypothesizes on limiting behavior.
- The student chooses and explains appropriate algebraic plan.
- The student justifies the application of L’Hôpital’s Rule.
- The student correctly concludes if L’Hôpital’s Rule is applicable.
- The student reasons and predicts if a limit exists or not.
- The student writes complete analysis with correct notation.

3b. identify and evaluate improper integrals, including integrals over infinite intervals and integrals in which the integrand becomes infinite in the interval of integration. The student deduces on his/her own that a given integral is improper. The student rephrases the integral precisely using limits and develops and executes a plan for calculation. The student presents the whole process.*

Sample Tasks:

- The student analyzes the integrand for possible singularities.
- The student recognizes if the interval of integration includes singularities.
- The student recognizes if the integrand end-behavior affects the integral.
- The student rewrites improper integrals with limits.
- The student reasons and predicts if integral will converge or diverge.
- The student writes complete analysis with correct notation.
- The student compares like integrands for similar behavior.

4. Sequences and Series: Successful Calculus students can analyze objects defined as limits. Calculus students are developing a working relationship with the infinite. They are experiencing how properties of limiting objects can differ from objects cited in the defining process. Students’ language and communication is evolving, especially in its precision. Sequence and series present a platform for the analysis of limiting numbers and functions.

The successful Calculus student can:

4a. critically analyze and discuss numerically, graphically, algebraically, verbally, and in other relevant ways a sequence or series of numbers. The student understands the difference between convergence and the limiting value and can determine convergence by using appropriate tests.*

Sample Tasks:

- The student explains what it means for a sequence of numbers to converge.
- The student presents a sequence as a function from the whole numbers (or integer subset).

- The student creates a graph of a sequence.
- The student calculates sequential values at given indices.
- The student distinguishes sequences from series.
- The student illustrates a series as a sequence of partial sums.
- The student hypotheses on convergence.
- The student speculates on choice of convergence test and summarizes expectations.
- The student conducts convergence test, explicitly satisfying conditions, and quoting conclusion.
- The student expands convergence concepts and procedures to include parameters and operate series as a function.
- The student separates questions of convergence from questions of limiting values.
- The student recognizes geometric series and can determine their limiting value.

4b. critically analyze and discuss numerically, graphically, algebraically, verbally, and in other relevant ways a sequence or series of functions, including Taylor and power series and associated error terms. The student understands the difference between convergence and the limiting function and can determine convergence by using appropriate tests. The student can apply the reasoning and techniques of Calculus with series representations *

Sample Tasks:

- The student explains what it means for a sequence of functions to converge to a function.
- The student presents a sequence of functions as a function from the whole numbers (or integer subset).
- The student creates a sequence of graphs for the component functions.
- The student evaluates functional values at given indices.
- The student distinguishes sequences from series.
- The student illustrates a series as a sequence of partial sums.
- The student debates properties of limiting function.
- The student illustrates properties of limiting function not present in any component/summand function.
- The student estimates domain and range of limiting function.
- The student hypothesizes on interval of convergence.
- The student creates nth Taylor polynomial.
- The student calculates error terms and expresses their meaning.
- The student compares Taylor series graphically.
- The student conducts convergence test and states domain of limiting function.
- The student moves the center of expansion and identifies the radius of convergence interval.
- The student creates series expansions around centers using existing series.

- The student explains a Taylor series that does not converge to the original function.
- The student differentiates limiting function.
- The student integrates limiting function.

5. Curves: Successful Calculus students understand the difference between curves and functions. Students heading to multi-variable calculus as well as STEM fields will encounter functions using ordered tuples as domain elements. Students must be able to change their perspective on curves depending on the situation. Sometimes students should interpret a tuple component as function information and use these for function analysis. Sometimes it is not intended that the components possess an independent-dependent variable relationship. The curve is a collection of points with geometric properties. Further, these points may have been collected via auxiliary functions and may be used as domain information for multi-variable functions. The successful Calculus student can:

5a. think parametrically. The student interacts with graphs with different perspectives depending on the situation.*

Sample Tasks:

- The student draws curve described parametrically.
- The student understands the difference between a curve and a parameterization.
- The student traces locations based on parameter behavior.
- The student calculates parameter values associated to curve properties.
- The student draws a curve with xy-axes or with other coordinate axes.
- The student converts coordinates from one system to another.
- The student theorizes on modifications to reverse parameterization.
- The student hypothesizes on modifications to shift parameterization.
- The student explains how to parameterize a line in a direction with an initial point.
- The student proposes parameterization for a given ellipse.
- The student can analyze curves expressed with polar coordinates.
- The student can convert coordinates between systems.

5b. measure area of bounded planar regions. Given a planar region whose boundary curves are described by equations, the student describes the situation in terms of functions and accompanying integration setup.*

Sample Tasks:

- The student draws boundary and shades region in given coordinate system.
- The student identifies limits of integration with respect to coordinate system.
- The student symbolizes area measurement with a definite integral.
- The student calculates area with a definite integral.

TMM010 – INTRODUCTORY STATISTICS *(Updated December 8, 2015; Updated with 2016 GAISE Guideline Recommendations, September 27, 2017)*

Typical Range 3-4 Semester Hours

This description is intended to apply to a range of introductory courses, from highly conceptual to more traditional presentations. It is assumed that technology is used (calculators, computer packages, or web application software) to minimize involved computations.

This is a course of study that stresses conceptual understanding and critical thinking and introduces statistical methods to college students in all disciplines. The American Statistical Association has developed a set of six recommendations for the teaching of introductory statistics – these recommendations are known as the "Guidelines for Assessment and Instruction in Statistics Education (GAISE)," which are strongly recommended in teaching or developing the introductory statistics course. The 2016 recommendations are as follows:

1. Teach statistical thinking.
    - Teach statistics as an investigative process of problem-solving and decision-making.
    - Give students experience with multivariable thinking.
2. Focus on conceptual understanding.
3. Integrate real data with a context and purpose.
4. Foster active learning.
5. Use technology to explore concepts and analyze data.
6. Use assessments to improve and evaluate student learning.

To qualify for TMM010 (Introductory Statistics), a course must achieve all of the following essential learning outcomes listed in this document (marked with an asterisk). The Sample Tasks are recommendations for types of activities that could be used in the course.

The successful Introductory Statistics students should be able to:

1. Summarize univariate and bivariate data by employing appropriate graphical, tabular, and numerical methods and describe the attributes of or relationships between the data. These may include (but are not limited to): frequency distributions; box plots; scatter plots; correlation coefficients; regression analysis; and measures of center, variation, and relative position.*

Sample Tasks:

- Distinguish between different types of data, such as categorical vs. quantitative variables and ordinal vs. nominal variables.

- Use and interpret graphical methods for summarizing data sets, such as box plots, histogram, and bar charts.
- Use and interpret numerical methods for summarizing data, such as mean, median, standard deviation, standardized scores, interquartile range, and relative frequency distribution.
- Assess descriptive methods that are most appropriate for highlighting specific features of data.
- Determine the relationship among mean, median and mode based on the shape of various datasets.
- Given bivariate data, choose appropriate graphical representation in order to determine whether a relationship exists between the variables, describe the nature of the relationship, and draw conclusions based on this relationship.
- Interpret the slope of a regression line in the context of the data.
- Make a comparison between datasets by interpreting appropriate data summaries.

2. Identify the characteristics of a well-designed statistical study and be able to critically evaluate various aspects of a study. Recognize the limitations of observational studies and common sources of bias in surveys and experiments. Recognize that association is not causation.*

Sample Tasks Given a research study, the student can:

- Distinguish between an observational study and an experimental study and discuss the advantages and disadvantages of each;
- Identify variables, the population of interest and the sampling technique;
- Compare various sampling techniques and the advantages and disadvantages of each;
- Identify possible sources of bias and confounding variables;
- Give several reasons why the results of this study would be challenged; and
- Interpret correlation vs. causation.

3. Compute the probability of compound events, independent events, and disjoint events, as well as conditional probability. Compute probabilities using discrete and continuous distributions, especially applications of the normal distribution.*

Sample Tasks:

- Compute and interpret an appropriate probability to answer probability question, make decisions, and justify conclusions.
- Distinguish between discrete and continuous distributions.
- Compute probabilities of events, unions of events, intersection of events, and conditional events in the context of two-way tables.
- Distinguish disjoint events from independent events.

- Check for independence of two events, either by computation using the definition or by reasoning based on the design of the experiment; determine when each method is most appropriate.
- Identify situations when it is necessary to compute conditional probabilities; compute conditional probabilities using either the definition or reasoning based on the experiment design; and determine when each method is most appropriate.
- Interpret the area under the density curve for continuous distribution and use it to approximate probabilities or proportions.

4. Explain the difference between statistics and parameters, describe sampling distributions, and generate sampling distributions to observe the Central Limit Theorem.* ,

Sample Tasks:

- Distinguish between the sampling distribution and the population distribution.
- Describe the terms “statistic” and “sampling variability”.
- Determine mean and standard deviation of the sampling distribution of a statistic.
- Describe how sample size affects the sampling distribution.
- Use the Central Limit Theorem in approximating distributions, such as approximating distribution of sample mean, distribution of sample proportion, or Binomial distribution.
- Generate or simulate sampling distributions to observe, empirically, the Central Limit Theorem.

5. Estimate population parameters using point and interval estimates and interpret the interval in the context of the problem. Summarize the relationship between the confidence level, margin of error, and sample size.*

Sample Tasks:

- Given a research objective and raw sample data, the student can: o Choose a proper confidence interval estimation method; o Verify assumptions behind the estimation method; and o Report and interpret confidence interval and margin of error.
- Evaluate the accuracy of sample estimates using standard errors.
- Explain how sample size and confidence level affect margin of error in the estimation.
- Determine the appropriate sample size for a specific margin of error and confidence level.

6. Given a research question, formulate null and alternative hypotheses. Describe the logic and framework of the inference of hypothesis testing. Make decision using p-value and draw appropriate conclusion. Interpret statistical significance and recognize that statistical significance does not necessarily imply practical significance. Perform hypothesis testing with at least one test related to quantitative variable (e.g. t-test for mean, test for linear correlation) and at least one test related to qualitative variable (e.g., test for one population proportion and chi-square test for independence).*

Sample Tasks:

- Given a research objective and raw sample data, the student can: o Translate research question or claim into null and alternative hypotheses; o Choose a proper hypothesis test; o Verify assumptions behind the test; and o Use p-value to interpret the statistical significance, make decision, and draw proper conclusion.
- Describe the logic and framework of hypothesis testing.
- Explain and demonstrate the effect of sample size in testing the statistical significance.
- Relate Type I Error and level of significance for the test when making decision.
- Interpret the statistical and practical significance.
- Distinguish between estimation problems and hypothesis testing problems.

7. Throughout this course, students should be given the opportunity to interpret statistical results in context when statistical information is presented in news stories and journal articles.*

Sample Tasks:

- Given a research study reported in either news story or journal article, the student can critically evaluate the study report, such as:
- Identifying the relevant population, sample, study units, variables of interest, and sampling method used;
- Distinguishing between observational and experimental study;
- Recognizing whether the study design permits conclusion about causation;
- Interpreting the p-value and confidence interval in context; and
- Describing possible biases in the data collection process.

TMM011 – QUANTITATIVE REASONING *(Endorsed December 21, 2015)*

Typical Range: 3-4 Semester Hours

Recommendation: This course should significantly reflect the Mathematical Association of America's Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide 2004:

"Engage students in a meaningful intellectual experience. Students must learn with understanding, focusing on relatively few concepts but treating them in depth. Treating ideas in depth includes presenting each concept from multiple points of view and in progressively more sophisticated contexts. For example, students are likely to improve their understanding more by writing analyses of a single situation that combines two or three mathematical ideas than by solving half a dozen problems using each idea separately.

Increase students' quantitative and logical reasoning abilities. Departments should encourage and support their institutions in establishing a quantitative literacy program for all students, with the primary goal of developing the intellectual skills needed to deal with quantitative information as a citizen and in the workplace. This program should ensure that all introductory and general education mathematics courses make a significant contribution toward increasing students' quantitative reasoning abilities . . . Students in these courses should also have the opportunity to use a variety of mathematical strategies—seeking the essential, breaking difficult questions into component parts, looking at questions from various points of view, looking for patterns—in diverse settings.

Improve students' ability to communicate quantitative ideas. "A Collective Vision: Voices of the Partner Disciplines" reports that nearly every discipline promotes the importance of having students communicate mathematical and quantitative ideas—both orally and in writing.

Encourage students to take other courses in the mathematical sciences. On the one hand, general education courses provide the last formal mathematics experience for most students and so must stand on their own intrinsic merits. On the other hand, they should be designed to serve as gateways and enticements for other mathematics courses.
Strengthen mathematical abilities that students will need in other disciplines. As reported in "A Collective Vision: Voices of the Partner Disciplines," faculty representing other disciplines emphasized the importance of mathematical modeling. Students should be able to create, analyze, and interpret basic mathematical models from informal problem statements; to argue that the models constructed are reasonable; and to use the models to provide insight into the original problem."

Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide 2004. Please see the entire 2004 CUPM for the complete set of recommendations being made 2 by the MAA on courses for non-STEM majors, which forms the basis for the outcome choices made herein.

An updated CUPM was released in 2015. What follows below is information from its introduction.

"The previous CUPM Curriculum Guide was published in 2004; it was followed by several related publications. Among them we call particular attention to the work of the CUPM subcommittee on Curriculum Renewal Across the First Two Years (CRAFTY) on the Curriculum Foundations Project, which emphasized the use of mathematics in other disciplines, and informs this Guide. Recommendations in this Guide reflect CUPM's reaffirmation of the principles in the 2004 Guide. Those principles, approved by MAA's Board of Governors in 2003 and reapproved in August 2014 can be found online. The 2004 Guide addressed the full range of mathematics offerings, including general education, service, and major courses. This Guide does not systematically address these non-major courses. This Guide focuses specifically on the design of mathematics majors, addressing the curricular demands of the wide and widening variety of mathematics programs now found across the nation. That diversity often leads to minors, concentrations, double majors, and interdisciplinary majors, as well as full majors in new and developing mathematically rich fields. The purpose of this Guide is to help departments adapt their undergraduate curricula to this changing landscape while maintaining the essential components of the traditional mathematics major."

The primary student population of a Quantitative Reasoning course is typically students seeking a Bachelor's of Arts degree requiring a 'liberal-arts' mathematics course. In addition, many other majors such as Nursing, Economics, and those requiring basic chemistry would benefit from such a course offering being included in their program of study.

As such, a Quantitative Reasoning course needs to highly emphasize the core mathematical general education outcome, critical thinking, as its primary objective and outcome. Through class discussion and working together in small groups, students can be facilitated in the development of the following core Quantitative Reasoning outcomes referenced in the 2015 CUPM:

1. "Interpretation: Ability to glean and explain mathematical information presented in various forms (e.g., equations, graphs, diagrams, tables, words).
2. Representation: Ability to convert information from one mathematical form (e.g., equations, graphs, diagrams, tables, words) into another.
3. Calculation: Ability to perform arithmetical and mathematical calculations.
4. Analysis/Synthesis: Ability to make and draw conclusions based on quantitative analysis.
5. Assumptions: Ability to make and evaluate important assumptions in estimation, modeling, and data analysis.
6. Communication: Ability to explain thoughts and processes in terms of what evidence is used, how it is organized, presented, and contextualized." Boersma, S., Diefenderfer, C. L., Dingman, S. W., & Madison, B. L. (2011). Quantitative reasoning in the contemporary world.

To qualify for TMM011 (Quantitative Reasoning), a course must achieve all of the following essential learning outcomes listed in this document (marked with an asterisk). The Sample Tasks are recommendations for types of activities that could be used in the course.

The successful Quantitative Reasoning student should be able to demonstrate these competencies:

1. NUMERACY: Students will develop and use the concepts of numeracy to investigate and explain quantitative relationships and solve problems in a variety of real-world contexts. (The teaching of numeracy is intended to both deepen and broaden understanding achieved in K-12, keeping the development and use of all but the most basic algebraic procedures to a minimum. One strategy to help students to deepen their knowledge and understanding is to require students to explain their thinking verbally and/or in writing and to receive feedback as they solve problems. Peer group discussions and reports may be a useful way to achieve this goal. Problems requiring rote use of arithmetic or algebraic procedures should be deemphasized, except when these procedures are essential to gaining deep conceptual understanding. Traditional textbook word problems are often artificial. In this course, realistic problems from a variety of realistic situations should be presented for discussion and solution. Technology should be used wherever appropriate.)

1.1 Solve real-world problems requiring the use and interpretation of ratios in a variety of contexts: Parts to whole comparisons, converting decimals to percentages and vice versa, quantifying risks by calculating and interpreting probabilities, rates of change, and margins of error.*

Sample Tasks:
a. Calculate income taxes using federal tax tables.
b. Interpret a news release of a medical intervention study mathematically by building tables with which false negatives and positives, sensitivity and specificity, can be analyzed to draw conclusions.
c. Given an excerpt concerning declining newspaper print readership, calculate and report on both the percentage point decline and the percentage decline.

1.2 Solve real-world problems relating to rates of change, distinguishing between and utilizing models that describe absolute change and relative change including growth and decay.*

Sample Tasks:
a. Demonstrate the correct use of absolute differences, reference values and relative changes to compare the rates of depreciation of select autos.
b. Interpret, analyze and explain a U.S. census report about smoking and distinguish between quantitative data referring to absolute changes and relative changes.
c. Describe the relationship between absolute change and linear models and the relationship between relative change and exponential models.

1.3 Compare and contrast statements which are proportional and those that are not by applying proportional reasoning appropriately to real-world situations such as scaling, dimensional analysis and modeling.*

Sample Tasks:
a. Decide if a real-world situation, such as the conversion of currencies or between systems of measurement, is a proportional relationship and then, if applicable, convert between currencies or systems.
b. Determine if the weight of a substance is proportional to its volume and compare that to whether the population of an area is proportional to the size of the area.
c. In general, recognize in a contextual setting the need for dimensional analysis and the ability to apply proportional reasoning to calculate such changes in representation.

1.4 Demonstrate numerical reasoning orally and/or by writing coherent statements and paragraphs.*

Sample Task:

- Student proficiency in this area would be demonstrated by the ability to communicate specific and complete information in such a way that the reader or listener can understand the contextual and quantitative information in a situation in addition to whatever quantitative and/or qualitative conclusions have been drawn.

2. Mathematical Modeling: Students will make decisions by analyzing mathematical models, including situations in which the student must recognize and/or make assumptions. (The teaching of mathematical modeling is intended to both deepen and broaden understanding achieved in K-12, keeping the development and use of all but the most basic algebraic procedures to a minimum. One strategy to help students to deepen their knowledge and understanding is to require students to explain their thinking verbally and/or in writing and to receive feedback as they solve problems. Peer group discussions and reports may be a useful way to achieve this goal. Problems requiring rote use of algebraic procedures should be de-emphasized, except when these procedures are essential to gaining deep conceptual understanding. Traditional textbook word problems are often artificial. In this course, realistic problems from a variety of realistic situations should be presented for discussion and solution. Technology should be used wherever appropriate.)

2.1 Create and use tables, graphs, and equations to model real-world situations including: using variables to represent quantities or attributes, estimating solutions to real-world problems using equations with variables, identifying pattern behavior, identifying how changing parameters can affect results, and identifying limitations in proposed models.*

Sample Tasks:
a. Create a table and graph depicting a linear relationship between a loan or investment monthly interest amount and the balance owed or due at the beginning of the month.
b. Describe in words an exponential model of population growth referring to the change as a 'constant rate of change'.
c. Accurately describe the limitations of a model of growth (e.g., of a population) or decline (e.g., of the Social Security Trust Fund) over a long period of time. d. Create an equation to model periodic compound interest and its relation to credit card balances.

2.2 Model financial applications such as credit card debt, installment savings, loans, etc. and calculate income taxes.*

Sample Tasks:
a. Use a spreadsheet to model personal savings growth and use it to find patterns of behavior based on national economic growth rates over periods of years.
b. Use a spreadsheet to forecast retirement savings growth based on varying long term assumptions; determine the length of time needed to achieve a personal savings goal.
c. Explain the limitations of mathematical models and risks in extrapolating information.

2.3 Create basic linear and exponential models for real-world problems and be able to choose which one is most appropriate for a given context and describe the limitations of the proposed models.* (Student proficiency in this area would be demonstrated by beginning with a real-world situation, deciding on the mathematical model and the form (graph, equation etc.), building the model, and finding and reporting on solutions and limitations.)

Sample Tasks:
a. Use a spreadsheet to model personal savings growth both with a regression line and an exponential curve and accurately assess the validity of both choices.
b. Make a decision to build an exponential model for growth or decay in a contextual situation based on quantitative information such as a constant percentage rate of change. Create the model.
c. Make a decision to build a linear model for growth or decay in a contextual situation based on quantitative information such as a constant absolute change. Create the model.

2.4 Use basic logarithm properties to address questions (regarding time periods etc.) arising in real-world situations modeled exponentially.*

Sample Tasks:
a. Use logarithmic properties to find the time required to achieve a personal savings goal.
b. Use logarithmic properties to find the time required for savings being compounded periodically to double.
c. Use logarithmic properties to find the half-life of models of decay.

2.5 Explain and critique models orally and/or by writing coherent statements and paragraphs.*

Sample Task:

- Student proficiency in this area would be demonstrated by the ability to communicate specific and complete information in such a way that the reader or listener can understand the contextual and quantitative information in a situation in addition to whatever quantitative and/or qualitative conclusions have been drawn.

3. Probability and Statistics: Students will use the language and structure of statistics and probability to investigate, represent, make decisions, and draw conclusions from real-world contexts.

(The teaching of statistics is intended to both deepen and broaden understanding achieved in K-12, keeping the development and use of all but the most basic statistical procedures to a minimum. One strategy to help students to deepen their knowledge and understanding is to require students to explain their thinking verbally and/or in writing and to receive feedback as they solve problems. Peer group discussions and reports may be a useful way to achieve this goal. Problems requiring rote use of statistical procedures should be de-emphasized, except when these procedures are essential to gaining deep conceptual understanding. Traditional textbook word problems are often artificial. In this course, realistic problems from a variety of realistic situations should be presented for discussion and solution. Technology should be used wherever appropriate.)

3.1 Critically evaluate statistics being presented in the media, journals, and other publications including evaluating the research methodology, critiquing how the author(s) came to their conclusions, identifying sources of bias, and identifying confounding variables. Students will be able to critically evaluate sampling strategy, the impact of sample size, correlation versus causation, and any inferences made.*

Sample Tasks:
a. Identify whether a news media article is reporting on a 'random controlled experiment' or an observational study to decide if conclusions being made are correlational or causational. Use this identification to discuss any inferences being made.
b. Identify possible confounding factors in a statistical study.
c. Identify possible biases in a statistical study. d. Critique media reports related to margin of error and statistical significance.

3.2 Summarize and interpret datasets with regard to shape, center, and spread. Use both graphical and numerical information. Use statistics appropriate to the shape. Students will be able to compare two or more datasets in light of this type of information.*

Sample Tasks:
a. In varied contextual situation (e.g., housing values, household income, demographic information), decide, with explanation, which of mean, median, and mode is the best measure of the 'center' set of a data set; compute the numerical values and explain their significance within the contexts.
b. Calculate basic measures of variation in a data set and explain why variation is an important concept in analyzing a set.
c. In a contextual situation, explain a reported standard deviation as the spread of the data around the mean.

3.3 Create visual representations of real-world data sets such as charts, tables, and graphs and be able to describe their strengths, limitations, and deceptiveness.*

Sample Tasks:
a. Create and use visual displays to analyze the variation and distribution of real-world data sets on crime data and to demonstrate various ways that quantitative information can be communicated or obfuscated.
b. Identify key factors such as vertical scaling to evaluate the accuracy and/or possible deceptiveness of charts.

3.4 Calculate probabilities and conditional probabilities in real-world settings, and employ them to draw conclusions.*

Sample Tasks:
a. Estimate probability in a contextual situation using data from a news article or a table.
b. Analyze a question such as "is it safer to drive that fly?" using probabilities.
c. Use probability to assess the real world trade-offs the modern world makes when making decisions such as driving rather than walking.

3.5 Justify decisions based on basic statistical (probabilistic) modeling orally and/or by writing coherent statements and paragraphs.*

Sample Task:
- Student proficiency in this area would be demonstrated by the ability to communicate specific and complete information in such a way that the reader or listener can understand the contextual and quantitative information in a situation in addition to whatever quantitative and/or qualitative conclusions have been drawn.

TMM013 – Business Calculus *(Updated June 3, 2015)*

Typical Range: 5-6 Semester Hours

In a Business Calculus course, students should:
- develop mathematical thinking and communication skills and learn to apply precise, logical reasoning to problem solving.
- be able to communicate the breadth and interconnections of the mathematical sciences through being presented key ideas and concepts from a variety of perspectives, a broad range of examples and applications, connections to business and other subjects, and contemporary topics and their applications.
- experience geometric as well as algebraic viewpoints and approximate as well as exact solutions.
- use computer technology to support problem solving and to promote understanding, as most calculus students, especially those who may take only one semester, profit from the use of a graphing utility and a tool for numerical integration.
  – Adapted from the MAA/CUPM 2004 Curriculum Guide

To qualify for TMM 013 (Business Calculus), a course must cover as a minimum the essential learning outcomes, noted by an asterisk. A course in Business Calculus may also commonly include some of the listed nonessential learning outcomes. These optional topics should be included only if there is adequate course time to do so beyond giving primary course attention to the essential learning outcomes. At least 70% of the classroom instructional time has to be spent on the essential learning outcomes. The optional learning outcomes are learning experiences that enhance, reinforce, enrich or are further applications of the essential learning outcomes. If review of prerequisite course content is necessary, only a minimal amount of time should be devoted to such review.

The successful Business Calculus student should be able to apply the following competencies to a wide range of functions, including piecewise, polynomial, rational, algebraic, exponential and logarithmic:

1. Demonstrate an understanding of limits and continuity:

1.01 Determine limits analytically, numerically and graphically including one-sided limits and limits at infinity.*

1.02 Analyze the limit behavior of a function at a point in its domain to determine if the function is continuous at that point. Determine intervals in which a function is continuous. Analyze and classify the discontinuities of a function.*

2. Demonstrate an understanding of derivatives and the ability to compute derivatives:
2.01 Use the limit definition of the derivative to determine the existence and to find the derivative of a given function.*

2.02 Find the derivative of a function by identifying and applying the appropriate derivative formula.*

2.03 Find higher order derivatives.*

3. Understand the interpretation of derivatives and their applications in a business environment:

3.01 Interpret the derivative as a rate of change.*

3.02 Find the slope of the tangent line to the graph of a function at a given point.*

3.03 Use the first derivative to determine intervals on which the graph of a function is increasing or decreasing and to determine critical points of the function.*

3.04 Use the second derivative to determine intervals on which the graph of a function is concave upwards or concave downwards and to determine points of inflection.*

3.05 Find and classify relative extrema and, on a closed interval, absolute extrema of a function.*

3.06 Solve applied problems including marginal analysis applications.*

3.07 Explain the relationship between marginal cost and average cost.*

3.08 Determine and discuss the elasticity of demand for a product.

4. Understand the concept of integration and demonstrate ability to find indefinite and definite integrals apply those results to the business setting:

4.01 Construct antiderivates analytically.*

4.02 Find indefinite integrals using integration formulas and the method of substitution.*

4.03 Find indefinite integrals using integration by parts.

4.04 Identify definite integrals of functions as the areas of regions between the graph of the function and the x-axis.*

4.05 Estimate the numerical value of a definite integral using a Riemann sum.

4.06 Understand and use the Fundamental Theorem of Calculus to evaluate definite integrals.*

4.07 Use definite integrals to calculate the area of the region under a curve and the area of the region between two curves.*

4.08 Determine present value and future value for an investment with interest compounded continuously.*

4.09 Determine the average value of a function on an interval.

4.10 For given supply and demand functions find and interpret the consumer's surplus and the producer's surplus.*

5. Demonstrate an understanding of functions of two variables:

5.01 Find the domain of a function of two variables.

5.02 Interpret contour diagrams for functions of two variables.

5.03 Compute partial derivatives of functions of two variables algebraically.

5.04 Determine critical points for functions of two variables.

5.05 Use the second derivative test to determine the nature of critical points of a function of two variables.

5.06 Use the method of Lagrange multipliers to determine extreme values of functions of two variables subject to constraints.

5.07 Solve applied problems involving the Cobb-Douglas production functions.

TMM014 – TECHNICAL MATHEMATICS I *(updated May 6, 2020)*

Typical Range: 4-5 Semester Hours

A course in Technical Mathematics specializes in the application of mathematics to the engineering technologies. The course emphasizes critical thinking by placing students in problem-solving situations and supporting students as they learn to make decisions, carry out plans, and judge results. Students encounter contextualized situations where concepts and skills associated with measurement, algebra, geometry, trigonometry, and vectors are the pertinent tools. The course highlights the supporting algebraic and analytical skills.

As a mathematics course in the applied fields, students studying Technical Mathematics 1 (TMM014) will benefit from more active and collaborative learning. Instead of extensive lectures dominating the presentation of skills and procedures, we hope this course will place students in situations where the mathematics become the active tools for investigation. Applications should be the foundation of a collaborative experience, where groups of students make decisions, choose tools, follow plans, draw conclusions, and explain their reasoning.

To qualify for TMM014 (Technical Mathematics 1), a course must achieve all of the following essential learning outcomes listed in this document (marked with an asterisk). The Illustrations exemplify the level of student engagement motivating this course.

1. Geometry: Successful Technical Mathematics students are able to visualize and identify geometric aspects and relationships within a given situation. Students can use these relationships to disseminate measurements throughout the situation. By reasoning with geometric properties, students can target specific information and explain their thought process.

The successful Technical Mathematics student can:

1a. recognize and apply properties of 2D shapes. Students recognize shapes within diagrams and use shapes in creating representations of situations. By selecting appropriate aspects and characteristics of the geometric shapes, students can extend a situational description via related lengths and areas. Through similar shapes and classifications, students explain their approach to problem solving. *

1b. recognize and apply properties of 3D shapes. Students visualize 3D situations and navigate around the picture or diagram. Through the use of common 3D shapes, students calculate volumes need for further calculations. *

1c. recognize and apply properties of angles. Student measures angles in degrees, radians, and DMS. Angular relationships, together with obtained or given angular measurements, are combined by student to elaborate about further angular measurements. *

1d. apply the Pythagorean Theorem. The student can identify right angle triangles in a diagram and attribute given measurements to the proper aspects. With this information, the student can use the Pythagorean Theorem to deduce other measurements. *

2. Measurement: Successful Technical Mathematics students are comfortable with measurements and their representations. Students anticipate types and classifications of measurements and use their expectations to formulate lines of investigation. Students obtain, convert, compare, and combine measurements. They express measurements in a variety of ways.

The successful Technical Mathematics student can:

2a. reason with amount measurements. Students distinguish qualities, characteristics, and aspects of objects in situations that can be measured. They connect units with appropriate measurement type. They convert units, and abbreviations, for similar types of measurements and can properly use units when discussing measurements. *

2b. can communicate about measurements. Students communicate fluently with units, abbreviations, and notation whether this be verbally or in writing. They calculate accurately with decimals, fractions, percentages, scientific notation, and engineering notation. They do so with information they have deciphered from tables and graphs. *

3. Equations, Inequalities, & Graphs: Successful Technical Mathematics students are proficient at algebraic procedures and manipulating measurements via equations, inequalities, and graphs. Students can express dependence and independence via equations and calculate resulting measurements via formulas. These calculations might be numeric, but more than likely, students should feel comfortable with the algebra of measurements.

The successful Technical Mathematics student can:

3a. algebraically manipulate expressions and equations and inequalities. Students are fluent in the use of arithmetic and algebra rules, especially fractions. They predict effects of changes in variables. They solve equations for dependent variables. They model situations with linear and quadratic equations. *

3b. combine algebraic and graphical representations. Students are proficient with all representations used to describe information and measurements. In particular, students competently use lines and linear equations to describe real-world relationships. *

3c. solve systems of equations. Students can create systems of linear equations from given situations and then solve them via various techniques. *

4. Dimensional Analysis: Successful Technical Mathematics students are proficient at combining measurements. Dimensions simply refers to products of measurements, which naturally emerge when working with rates. Students reason through the connecting changes described by the rates and manipulate units symbolically via exponential forms.

The successful Technical Mathematics student can:

4a. reason with rate measurements. Students express rates as phrases, equations, and fractions. They compare the changing amounts described by rates. They can convert units and reason for the needed rate. *

4b. manipulate algebraic representations of amounts and rates. Students algebraically manipulate rates written as fractions. They can reduce and simplify products of rate fractions using the properties of exponents. *

5. Trigonometry: Successful Technical Mathematics students demonstrate an ability to use triangles. Students generate a triangle mesh as needed to combine given information together and propagate a chain of measurements toward a desired target.

The successful Technical Mathematics student can:

5a. identify right triangles in situations. Students identify right triangular configurations in diagrams and assign given measurements to the proper pieces of the triangles. *

5b. apply trigonometric functions. Students identify which pieces of a right triangle are given and apply sine, cosine, and tangent functions to obtain further measurements. They use calculators to evaluate expressions involving trigonometric functions. They apply the Laws of Sine and Cosine appropriately. *

5c. apply inverse trigonometric functions. Students recognize if questions are asking for lengths or angle measurements and can phrase a solution plan. Using calculators, students can obtain angular measurements from linear measurements. *

5d. analyze and compare basic sine and cosine graphs. Students can graphically determine amplitude, period, phase shift, etc. *

6. Vectors: Successful Technical Mathematics students are proficient with vectors. Students rephrase information in terms of vectors and easily moving between components and resultants.

The successful Technical Mathematics student can:

6a. represent vectors. Students represent vectors algebraically and graphically in either rectangular or polar coordinates. *

6b. perform arithmetic with vectors. Students represent vectors algebraically and graphically and perform arithmetic operations in either mode. *

6c. Decompose vectors. Students decompose measurements into components relative to given coordinate systems. *

7. Complex Numbers: Successful Technical Mathematics students are comfortable with the arithmetic of complex numbers as well as their basic Geometry.

The successful Technical Mathematics student can:

7a. introduce arithmetic with complex numbers. Add, subtract, multiply, and divide. *

7b. connect algebraic expressions with points in the plane. $a + bi$ associated with (a, b). *

7c. map complex arithmetic with vector arithmetic. *

8. Functions: Successful Technical Mathematics students are proficient with dependencies. Students analyze the effect on dependent quantities from changes in independent quantities, whether this dependency is described algebraically or graphically.

The successful Technical Mathematics student can:

8a. communicate via function notation. *

8b. evaluate functions via formulas or graphs. *

8c. obtain graphs via technology given formulas. *

8d. analyze function behavior graphically. (increasing/decreasing, maximums/minimums) *

8e. work explicitly with linear functions *

8f. work explicitly with quadratic functions. *

8g. evaluate, graph, and graphically analyze exponential functions. *

TMM015 – TECHNICAL MATHEMATICS II *(updated May 6, 2020)*

Typical Range: 4-5 Semester Hours

A course in Technical Mathematics specializes in the application of mathematics to the engineering technologies. The course emphasizes critical thinking by placing students in problem-solving situations and supporting students as they learn to make decisions, carry out plans, and judge results. Students encounter contextualized situations where concepts and skills associated with measurement, algebra, geometry, trigonometry, and vectors are the pertinent tools. The course highlights the supporting algebraic and analytical skills.

As a mathematics course in the applied fields, students studying Technical Mathematics 2 (TMM015) will benefit from more active and collaborative learning. Instead of extensive lectures dominating the presentation of skills and procedures, we hope this course will place students in situations where the mathematics become the active tools for investigation. Applications should be the foundation of a collaborative experience, where groups of students make decisions, choose tools, follow plans, draw conclusions, and explain their reasoning.

To qualify for TMM015 (Technical Mathematics 2), a course must achieve all of the following essential learning outcomes listed in this document (marked with an asterisk). The Illustrations exemplify the level of student engagement motivating this course.

1. Basic Elementary Functions: Successful Technical Mathematics students recognize basic function forms, algebraically and graphically, and can predict general behavior from this classification. Students can produce graphs of basic functions and provide descriptions of their behavior. Finally, students can develop and present this analysis as they deem useful.

The successful Technical Mathematics student can:

1a. analyze basic polynomial functions. From a factored form, students identify zeros, multiplicities, locate and classify corresponding graph intercepts. From the graph or factorization, students discern locations of possible local maximums and minimums. Students produce graphs of polynomials given in factored form and describe the connection between zeros, factors, and intercepts. By examining the factored form, students can predict polynomial end-behavior. *

1b. analyze basic rational functions. From a factored form, students identify zeros, poles, multiplicities, locate and classify corresponding graph intercepts and vertical asymptotes. From the graph or factorization, students discern locations of possible local maximums and minimums. Students produce graphs of rational functions given in factored form and describe the connection between zeros, factors, and intercepts and asymptotes. By examining the factored form, students can predict polynomial end-behavior and represent this graphically. *

1c. analyze basic exponential functions. Students recognize basic exponential forms from formulas and graphs. Students explain general function properties and behavior and produce graphs from these. *

1d. analyze basic logarithmic functions. Students recognize basic logarithmic forms from formulas and graphs. Students explain general function properties and behavior and produce graphs from these. *

1e. analyze basic roots and radical functions. Students describe general function properties and behavior and produce graphs from these. *

1f. analyze basic trigonometric functions. Students are fluent with the properties and behavior of sine, cosine, and tangent. Students identify zeros and their corresponding intercepts as well as asymptotes. Students provide exact values when appropriate and otherwise provide estimations. *

2. Algebraic Properties: Successful Technical Mathematics students apply basic algebraic properties of function types to produce equivalent forms appropriate for the current question or investigation.

The successful Technical Mathematics student can:

2a. apply the distributive property effectively. Students decide when to factor or expand expressions to support a particular goal or enhance communication. Students identify common factors and construct equivalent products when seeking zeros. *

2b. communicate fluently with the language of Algebra. Students view expressions from the details of individual components to the patterns in which those components are suspended. Students attend to the broad algebraic structure before addressing details. *

2c. manipulate exponents. Students condense, combine, and expand exponential expressions. *

2d. manipulate logarithms. Students condense, combine, and expand logarithmic expressions. Students convert exponential-logarithmic compositions. *

2e. manipulate radicals. Students condense, combine, and expand radical expressions. Students swap radical expressions for exponential expressions and vice-versa. *

2f. utilize trigonometric identities. Students can use basic trigonometric identities to construct equivalent expressions. *

3. Rate of Change: Successful Technical Mathematics students communicate effectively about situational rates as well as rates-of-change encoded within functions.

The successful Technical Mathematics student can:

3a. calculate rates of change over intervals. Students calculate rate-of-change and interpret. *

3b. linearize a function around a point. Students plot tangent lines to graphs of functions and create linear functions that approximate the original function. *

3c. represent rate-of-change as a function. Students sketch approximate graphs for rate-of-change of a function. Use these to describe function behavior. *

4. Composition: Successful Technical Mathematics students are comfortable with the operations of functions, especially composition.

The successful Technical Mathematics student can:

4a. compose functions. Students compose functions algebraically. Students can identify component functions from a given composition. Students can establish domains and ranges of compositions. *

4b. employ composition as an operation. Students view the identity function as the identity element in composition. Students can construct inverse functions and produce the identity function via composition. *

4c. express linear composition graphically. Students interpret linear compositions in terms of graphical transformations. Students encapsulate graphical transformations algebraically. *

5. Analytical Geometry: Successful Technical Mathematics students are comfortable with various descriptions and interpretations of curves, especially circles and ellipses.

The successful Technical Mathematics student can:

5a. connect Cartesian equations and graphs of ellipses. Students can plot ellipses from equations. Students can create equations from plotted ellipses. *

5b. parameterize ellipses. Students parameterize ellipses with sine and cosine.*

5c. parameterize lines. Students analyze multiple parameterizations for the same line. *

TMM017-CALCULUS I & II SEQUENCE *(Updated June 3, 2015)*

Typical Range: 8-10 Semester Hours

This sequence Ohio Transfer 36 is a combination of the learning outcomes in TMM005 (Calculus I) and TMM006 (Calculus II).

TMM005 -CALCULUS I

Typical Range: 4-5 Semester Hours

In a Calculus I course, students should:

- develop mathematical thinking and communication skills and learn to apply precise, logical reasoning to problem solving.

- be able to communicate the breadth and interconnections of the mathematical sciences through being presented key ideas and concepts from a variety of perspectives, a broad range of examples and applications, connections to other subjects, and contemporary topics and their applications.
- experience geometric as well as algebraic viewpoints and approximate as well as exact solutions.
- use computer technology to support problem solving and to promote understanding, as most calculus students, especially those who may take only one semester, profit from the use of a graphing utility and a tool for numerical integration.
- for students in the mathematical sciences, progress from a procedural/computational understanding of mathematics to a broad understanding encompassing logical reasoning, generalization, abstraction, and formal proof; gain experience in careful analysis of data; and become skilled at conveying their mathematical knowledge in a variety of settings, both orally and in writing. – Adapted from the MAA/CUPM 2004 Curriculum Guide

To qualify for Ohio Transfer 36 equivalency of TMM005 (Calculus I), a course must cover as a minimum the essential learning outcomes, denoted by an asterisk (*). A Calculus I course may also commonly include some of the listed nonessential learning outcomes. These optional topics should be included only if there is adequate course time to do so beyond giving primary course attention to the essential learning outcomes. At least 70% of the classroom instructional time has to be spent on the essential learning outcomes. The optional learning outcomes are learning experiences that enhance, reinforce, enrich or are further applications of the essential learning outcomes. If review of prerequisite course content is necessary, only a minimal amount of time should be devoted to such review.

The successful Calculus I student should be able to apply the following competencies to a wide range of functions, including piecewise, polynomial, rational, algebraic, trigonometric, inverse trigonometric, exponential and logarithmic:

1. Determine the existence of, estimate numerically and graphically and find algebraically the limits of functions. Recognize and determine infinite limits and limits at infinity and interpret them with respect to asymptotic behavior.*
2. Determine the continuity of functions at a point or on intervals and distinguish between the types of discontinuities at a point.*
3. Determine the derivative of a function using the limit definition and derivative theorems. Interpret the derivative as the slope of a tangent line to a graph, the slope of a graph at a point, and the rate of change of a dependent variable with respect to an independent variable.*
4. Determine the derivative and higher order derivatives of a function explicitly and implicitly and solve related rates problems.*

5. Determine absolute extrema on a closed interval for continuous functions and use the first and second derivatives to analyze and sketch the graph of a function, including determining intervals on which the graph is increasing, decreasing, constant, concave up or concave down and finding any relative extrema or inflection points. Appropriately use these techniques to solve optimization problems.*
6. Determine when the Mean Value Theorem can be applied and use it in proofs of other theorems such as L'Hopital's rule.
7. Use differentials and linear approximations to analyze applied problems.
8. Determine antiderivatives, indefinite and definite integrals, use definite integrals to find areas of planar regions, use the Fundamental Theorems of Calculus, and integrate by substitution.*

TMM006 - CALCULUS II *(Updated June 3, 2015)*

Typical Range: 4-5 Semester Hours

In a Calculus II course, students should:

- develop mathematical thinking and communication skills and learn to apply precise, logical reasoning to problem solving, as emphasized in the calculus renewal movement.
- be able to communicate the breadth and interconnections of the mathematical sciences through being presented key ideas and concepts from a variety of perspectives, a broad range of examples and applications, connections to other subjects, and contemporary topics and their applications.
- experience geometric as well as algebraic viewpoints and approximate as well as exact solutions.
- use computer technology to support problem solving and to promote understanding (e.g., technology allows students easy access to the graphs of planar curves and visualization helps students understand concepts such as approximation of integrals by Riemann sums or functions by Taylor polynomials; symbolic manipulation can be handled allowing students to focus their attention on understanding concepts; computer algorithms can be explored; and conjectures can be posited, investigated and refined, such as manipulating parameters on classes of functions and fitting functional models to data).
- for students in the mathematical sciences, progress from a procedural/computational understanding of mathematics to a broad understanding encompassing logical reasoning, generalization, abstraction, and formal proof; gain experience in careful analysis of data; and become skilled at conveying their mathematical knowledge in a variety of settings, both orally and in writing. – Adapted from the MAA/CUPM 2004 Curriculum Guide

To qualify for Ohio Transfer 36 equivalency of TMM006 (Calculus II), a course must cover as a minimum the essential learning outcomes, denoted by an asterisk (*). A Calculus II course may also commonly include some of the listed nonessential learning outcomes. These optional topics should be included only if there is adequate course time to do so beyond giving primary course attention to the essential learning outcomes. At least 70% of the classroom instructional time has to be spent on the essential learning outcomes. The optional learning outcomes are learning experiences that enhance, reinforce, enrich or are further applications of the essential learning outcomes. If review of prerequisite course content is necessary, only a minimal amount of time should be devoted to such review.

The successful Calculus II student should be able to:

1. Use antiderivatives to evaluate definite integrals and apply definite integrals in a variety of applications to model physical, biological or economic situations. Whatever applications (e.g. determining area, volume of solids of revolution, arc length, area of surfaces of revolution, centroids, work, and fluid forces) are chosen, the emphasis should be on setting up an approximating Riemann sum and representing its limit as a definite integral.*
2. Approximate a definite integral by the Trapezoidal Rule and Simpson's Rule.
3. Employ a variety of integration techniques to evaluate special types of integrals, including substitution, integration by parts, trigonometric substitution, and partial fraction decomposition.*
4. Evaluate limits that result in indeterminate forms, including the application of L'Hôpital's Rule.*
5. Evaluate improper integrals, including integrals over infinite intervals, as well as integrals in which the integrand becomes infinite on the interval of integration.*
6. Find, graph, and apply the equations of conics, including conics where the principal axes are not parallel to the coordinate axes.
7. Determine the existence of, estimate numerically and graphically, and find algebraically the limits of sequences. Determine whether a series converges by using appropriate tests, including the comparison, ratio, root, integral and alternating series tests.*
8. Find the n th Taylor polynomial at a specified center for a function and estimate the error term. Use appropriate techniques to differentiate, integrate and find the radius of convergence for the power series of various functions.*
9. Analyze curves given parametrically and in polar form and find the areas of regions defined by such curves.*
10. Perform and apply vector operations, including the dot and cross product of vectors, in the plane and space.
11. Solve separable differential equations. Understand the relationship between slope fields and solution curves for differential equations. Use Euler's method to find numerical solutions to differential equations.

OMT018- Calculus III 4-5 Semester Hours

Related TAGs: Chemistry, Computer/Electrical Engineering, Mathematics, Physics

Prerequisite: The prerequisite for Calculus III is generally TMM 006 Calculus II.

Statewide Learning Outcomes:

In a Calculus III course, students should:

- develop mathematical thinking and communication skills and learn to apply precise, logical reasoning to problem solving, as emphasized in the calculus renewal movement.
- be able to communicate the breadth and interconnections of the mathematical sciences through being presented key ideas and concepts from a variety of perspectives, a broad range of examples and applications, connections to other subjects, and contemporary topics and their applications.
- experience geometric as well as algebraic viewpoints and approximate as well as exact solutions.
- use computer technology to support problem solving and to promote understanding (e.g., graphics packages enhance multivariable calculus).
- for students in the mathematical sciences, progress from a procedural/computational understanding of mathematics to a broad understanding encompassing logical reasoning, generalization, abstraction, and formal proof; gain experience in careful analysis of data; and become skilled at conveying their mathematical knowledge in a variety of settings, both orally and in writing. -- Adapted from the MAA/CUPM 2004 Curriculum Guide

To qualify for TAG equivalency of OMT018 (Calculus III), a course must cover as a minimum the essential learning outcomes, denoted by an asterisk (*). A Calculus III course may also commonly include some of the listed nonessential learning outcomes. These optional topics should be included only if there is adequate course time to do so beyond giving primary course attention to the essential learning outcomes. At least 70% of the classroom instructional time has to be spent on the essential learning outcomes. The optional learning outcomes are learning experiences that enhance, reinforce, enrich or are further applications of the essential learning outcomes. If review of prerequisite course content is necessary, only a minimal amount of time should be devoted to such review.

The successful Calculus III student should be able to:

1. Perform and apply vector operations, including the dot and cross product of vectors, in the plane and space. Graph and find equations of lines, planes, cylinders and quadratic surfaces.*
2. Differentiate and integrate vector-valued functions. For a position vector function of time, interpret these as velocity and acceleration.*
3. Evaluate limits and determine the continuity and differentiability of functions of several variables. *
4. Describe graphs, level curves and level surfaces of functions of several variables.*
5. Find arc length and curvature of space curves, including the use of unit tangents and unit normals; identify and interpret tangential and normal components of acceleration.
6. Find partial derivatives, directional derivatives, and gradients and use them to solve applied problems.*

7. Find differentials of functions of several variables and use them to solve applied problems.
8. Find equations of tangent planes and normal lines to surfaces that are given implicitly or parametrically.*
9. Use the chain rule for functions of several variables (including implicit differentiation).*
10. For functions of several variables, find critical points using first partials and interpret them as relative extrema/saddle points using the second partials test. Find absolute extrema on a closed region. Apply these techniques to optimization problems.*
11. Use Lagrange multipliers to solve constrained optimization problems.
12. Evaluate multiple integrals in appropriate coordinate systems such as rectangular, polar, cylindrical and spherical coordinates and apply them to solve problems involving volume, surface area, density, moments and centroids.*
13. Use Jacobians to change variables in multiple integrals.
14. Evaluate line and surface integrals. Identify when a line integral is independent of path and use the Fundamental Theorem of Line Integrals to solve applied problems.*
15. Identify conservative and inverse square fields.*
16. Find the curl and divergence of a vector field, the work done on an object moving in a vector field, and the flux of a field through a surface. Use these ideas to solve applied problems.*
17. Introduce and use Green's Theorem, the Divergence (Gauss's) Theorem and Stokes's Theorem.*

OMT019 - ELEMENTARY LINEAR ALGEBRA

3-4 Semester Hours/4-5 Quarter Hours

Related TAGs: Math, Physics

In an Elementary Linear Algebra course, students should:

- develop mathematical thinking and communication skills and learn to apply precise, logical reasoning to problem solving (e.g., teaching computational techniques should not override the goal of leading students to understand fundamental mathematical relationships).
- be able to communicate the breadth and interconnections of the mathematical sciences through being presented key ideas and concepts from a variety of perspectives, a broad range of examples and applications, connections to other subjects, and contemporary topics and their applications.
- experience geometric as well as algebraic viewpoints and approximate as well as exact solutions.
- use computer technology to support problem solving and to promote understanding (e.g., linear algebra courses can use technology for matrix manipulation or for visualizing the effects of linear transformations in two or three dimensions, and technology makes large linear systems tractable).

- for students in the mathematical sciences, progress from a procedural/computational understanding of mathematics to a broad understanding encompassing logical reasoning, generalization, abstraction, and formal proof; gain experience in careful analysis of data; and become skilled at conveying their mathematical knowledge in a variety of settings, both orally and in writing.

    -Adapted from the MAA/CUPM 2004 Curriculum Guide

The prerequisite for an Elementary Linear Algebra course is generally either a year-long sequence in calculus (TMM005 Calculus I & TMM006 Calculus II) or OMT018 Calculus III. Adapted statements regarding Calculus in the MAA/CUPM 2004 Curriculum Guide also generally apply to Elementary Linear Algebra.

In addition, any elementary linear algebra course should

- respond to the needs of client disciplines;
- focus on a being a matrix-oriented course;
- maintain a strong geometric emphasis;
- consider the needs and interests of students as learners;
- utilize technology to reinforce concepts, contribute to discoveries, and support finding solutions to realistic applied problems; and
- lead to a second course in matrix theory/linear algebra.

    -Adapted from the 1993 Linear Algebra Curriculum Study Group Recommendations

To qualify for TAG equivalency of OMT019 (Elementary Linear Algebra), a course must cover as a minimum the essential learning outcomes, denoted by an asterisk (*). An Elementary Linear Algebra course may also commonly include some of the listed nonessential learning outcomes. These optional topics should be included only if there is adequate course time to do so beyond giving primary course attention to the essential learning outcomes. At least 70% of the classroom instructional time has to be spent on the essential learning outcomes. The optional learning outcomes are learning experiences that enhance, reinforce, enrich or are further applications of the essential learning outcomes. If review of prerequisite course content is necessary, only a minimal amount of time should be devoted to such review .

The successful Linear Algebra student should be able to:

1. **Understand algebraic and geometric representations of vectors in R *n* and their operations, including addition, scalar multiplication and dot product. understand how to determine the angle between vectors and the orthogonality of vectors.***
2. **Solve systems of linear equations using Gauss-Jordan elimination to reduce to echelon form. Solve systems of linear equations using the inverse of the coefficient matrix when possible. Interpret existence and uniqueness of solutions geometrically.***

3. **Perform common matrix operations such as addition, scalar multiplication, multiplication, and transposition. Discuss associativity and noncommutativity of matrix multiplication.***
4. **Discuss spanning sets and linear independence for vectors in R *n*. For a subspace of R *n*, prove all bases have the same number of elements and define the dimension. Prove elementary theorems concerning rank of a matrix and the relationship between rank and nullity.***
5. **Interpret a matrix as a linear transformation from R *n* to R *m*. Discuss the transformation's kernel and image in terms of nullity and rank of the matrix. Understand the relationship between a linear transformation and its matrix representation, and explore some geometric transformations in the plane. Interpret a matrix product as a composition of linear transformations.***
6. **Use determinants and their interpretation as volumes. Describe how row operations affect the determinant. Analyze the determinant of a product algebraically and geometrically.***
7. **Define eigenvalues and eigenvectors geometrically. Use characteristic polynomials to compute eigenvalues and eigenvectors. Use eigenspaces of matrices, when possible, to diagonalize a matrix.***
8. **Use axioms for abstract vector spaces (over the real or complex fields) to discuss examples (and non-examples) of abstract vector spaces such as subspaces of the space of all polynomials.***
9. Discuss the existence of a basis of an abstract vector space. Describe coordinates of a vector relative to a given basis. For a linear transformation between vector spaces, discuss its matrix relative to given bases. Discuss how those matrices changes when the bases are changed.
10. Discuss orthogonal and orthonormal bases, Gram-Schmidt orthogonalization, orthogonal complements and projections. Discuss rigid motions and orthogonal matrices.
11. Discuss general inner product spaces and symmetric matrices, and associated norms. Explain how orthogonal projections relate to least square approximations.

OMT020 - ELEMENTARY DIFFERENTIAL EQUATIONS

3-4 Semester Hours/4-5 Quarter Hours

Related TAGs: AACM Engineering, Chemical Engineering, Math, Physics
In an Elementary Differential Equations course, students should:

- develop mathematical thinking and communication skills and learn to apply precise, logical reasoning to problem solving, as emphasized in the calculus renewal movement.
- be able to communicate the breadth and interconnections of the mathematical sciences through being presented key ideas and concepts from a variety of perspectives, a broad range of examples and applications, connections to other subjects, and contemporary topics and their applications.

- experience geometric as well as algebraic viewpoints and approximate as well as exact solutions.
- use computer technology to support problem solving and to promote understanding (e.g., most modern texts make use of a differential equation solver that can permit the early introduction of modeling with systems of differential equations).
- for students in the mathematical sciences, progress from a procedural/computational understanding of mathematics to a broad understanding encompassing logical reasoning, generalization, abstraction, and formal proof; gain experience in careful analysis of data; and become skilled at conveying their mathematical knowledge in a variety of settings, both orally and in writing.

  -Adapted from the MAA/CUPM 2004 Curriculum Guide

The prerequisite for an Elementary Differential Equations course is generally either a year-long sequence in calculus (TMM005 Calculus I & TMM006 Calculus II) or OMT018 Calculus III. Since an Elementary Differential Equations course follows from differential and integral calculus, adapted statements regarding Calculus in the MAA/CUPM 2004 Curriculum Guide also apply to Elementary Differential Equations.

To qualify for OMT020 (Elementary Differential Equations), a course must cover as a minimum the essential learning outcomes, denoted by an asterisk *. A course in Elementary Differential Equations may also commonly include some of the listed nonessential learning outcomes. These optional topics should be included only if there is adequate course time to do so beyond giving primary course attention to the essential learning outcomes. At least 70% of the classroom instructional time has to be spent on the essential learning outcomes. The optional learning outcomes are learning experiences that enhance, reinforce, enrich or are further applications of the essential learning outcomes. If review of prerequisite course content is necessary, only a minimal amount of time should be devoted to such review .

The successful Elementary Differential Equations student should be able to:

1. Solve first-order differential equations that are separable, linear or exact. *
2. Solve first-order differential equations by making the appropriate substitutions, including homogeneous and Bernoulli equations.*
3. Use linear or nonlinear first-order differential equations to solve application problems such as exponential growth and decay, population logistics growth, velocity, solution mixtures, two component series circuits and chemical reactions.*
4. Understand the relationship between slope fields and solution curves for differential equations. Use a slope field and an initial condition to estimate a solution curve to a differential equation.*
5. Approximate solutions of first-order differential equations using Euler and Runge-Kutta methods.
6. Use the method of reduction of order.

7. Solve higher-order homogeneous linear equations with constant coefficients. *
8. Solve higher-order nonhomogeneous linear equations with constant coefficients by the method of undetermined coefficients. *
9. Solve higher-order nonhomogeneous linear equations by the method of variation of parameters. *
10. Use linear second-order differential equations to solve application problems such as spring/mass system motion problems, acceleration, or three component series circuits. *
11. Solve application problems requiring the use of higher-order differential equations with boundary conditions, such as the whirling string, the deflection of a uniform beam and the buckled rod.
12. Use power series to solve higher-order differential equations about ordinary or singular points.
13. Solve special classes of equations such as Cauchy-Euler, Bessel and Legendre equations.
14. Perform operations with Laplace and inverse Laplace transforms to solve higher-order differential equations.*
15. Solve systems of differential equations.

TMM021: Mathematics in Elementary Education I *(updated May 8, 2020)*

Typical Range: 4-5 Semester

Hours Recommendation:

It is essential for all teachers of mathematics to understand the reasoning underlying the mathematics they are teaching. They need to understand why various procedures work, how each idea they will be teaching connects with other important ideas in mathematics, and how these ideas develop and become more sophisticated. Furthermore, knowing only the mathematics of the elementary grades is not sufficient to be an effective teacher of elementary grades mathematics. Neither is it sufficient to require that future teachers simply “take more math courses.” This document describes the kinds of mathematics and mathematical experiences that we believe are essential for their mathematical learning and professional development. Exploration in middle grade topics are introduced at the elementary level to begin planting the seed in preparation for advanced teachings.

We take the view that mathematics courses for future teachers must prepare them to do a different kind of work in their mathematics teaching than they likely experienced in their own schooling. With this in mind, we encourage oral and written communication in these classes as both a learning tool and as preparation for handling mathematical questions which arise in their classrooms. For example, we aim for discussion that focuses on the deep mathematical reasoning underlying the computational procedures that are usually taught in elementary school. We recommend this be done by exploring common misconceptions with preservice teachers and by engaging future teachers in activities that require them to interpret their own multiple ways of addressing questions and interpreting children’s work which might be incorrect, incomplete, or different from their adult ways of thinking. We aim to encourage a serious approach to the mathematics through questions (by instructors *and* preservice teachers) about *why* we do things the way we do, what the operations *mean*, what the *units* are on the answers, and how the mathematical ideas of the day *connect* to other mathematical ideas (looking for the “big ideas” in a problem).

We recommend these courses be activity based so that opportunities for deep, connected learning arise while misconceptions are addressed. This requires “good” problems and “good” questioning by instructors. A good problem needs to engage the preservice teachers at an appropriate level of challenge (hard enough that the preservice teacher cannot answer on autopilot). Often this is accomplished by confronting them with misconceptions framed as “a child said this…” and directions to analyze and/or justify the result. The justifications can themselves become a source of deep discussion - one preservice teacher may not understand another’s solution or explanation, a preservice teacher’s correct answer may have a flawed explanation, or a new method may be generated once an array of other methods has been shared. Sometimes the discussion is generated when an instructor says, “I want to list all of the different answers we got before we discuss the reasoning” (which means the class can actually discuss their own wrong answers). All of the above takes a lot of time, so we recommend 8 to 10 credit hours of such work in teacher education programs or prerequisites.

At the same time, we strongly urge that instructors be aware that these are college level mathematics courses. “A credit-bearing, college-level course in Mathematics must use the standards required for high school graduation by the State of Ohio as a basis and must do at least one of the following: 1)broaden, or 2) deepen, or 3) extend the student’s learning.” We recommend dedicated coursework for this content because there is much deep mathematics to be explored in understanding what we loosely term “elementary school mathematics.” This is a mathematical content course. The learning outcomes are all focused on using, justifying, and connecting mathematical concepts and do not address “how to teach.” It is sometimes appropriate to discuss topics which are more directly relevant to a methods course when they serve the purpose of motivating a mathematical discussion, but students should not be assessed on methods in these courses.

The courses should integrate reasoning, flexibility, multiple explanations, and number sense. Leading questions help students to make connections among topics and to develop their own questioning skills (e.g. Have we seen this idea before? How are these two different solution methods related to each other? What does it mean? How do you know? Could you draw a picture to show it? Where did they go wrong? Could we use their idea in this other problem?) Students should understand that mathematics is correct if it makes personal sense and if one can explain it in a way to make sense to others, not if an authority certifies it. Preservice teachers should leave these courses knowing that math makes sense and armed with the underlying knowledge they need to make math make sense for their future students. Elementary students will often come up with their own creative approaches to problems, so future teachers must be able to evaluate their mathematical viability *before* deciding how to respond instructionally. Squashing a child’s idea can be quite harmful. And often children’s ideas are right or almost right.

The learning outcomes that follow are all viewed as essential by the committee (marked with an asterisk) and demonstrate the level of student engagement motivating this course. In addition, learning outcomes are not specific items/topics but rather learning outcomes of course entirety. Institutional courses should provide an integrated experience with learning outcomes woven together throughout the course.

1. **Numbers**
   The successful Mathematics in Elementary Education student can:

   1a. Discuss the intricacies of learning to count, including the distinction between counting as a list of numbers in order and counting to determine a number of objects, and use pairings between elements of two sets to establish equality or inequalities of cardinalities. *

   1b. Attend closely to units (e.g., apples, cups, inches, etc.) while solving problems and explaining solutions. *

   1c. Discuss how the base-ten place value system (including extending to decimals) relies on repeated bundling in groups of ten and how to use objects, drawings, layered place value cards, base-ten blocks, and numerical expressions (including integer exponents) to help reveal base-ten structure. *

   1d. Use the CCSS (Common Core State Standards) development of fractions: *
   - Start with a whole.
   - Understand the fraction 1/b as one piece when the whole is divided into b equal pieces.
   - Understand the fraction a/b as a pieces of size 1/b and that the fraction a/b may be larger than one.
   - Understand fractions as numbers that can be represented in a variety of ways, such as with lengths (esp. number lines), areas (esp. rectangles), and sets (such as a collection of marbles).
   - Use the meaning of fractions to explain when two fractions are equivalent.

   1e. Model positive versus negative numbers on the number line and in real-world contexts. *

   1f. Reason about the comparison (=, <, >) of numbers across different representations (such as fractions, decimals, mixed numbers, ...). *

   1g. Demonstrate the skill of calculating simple arithmetic problems WITHOUT the use of a calculator. *

2. Operations

The successful Mathematics in Elementary Education student can:

2a. Recognize addition, subtraction, multiplication, and division as descriptions of certain types of reasoning and correctly use the language and notation of these operations. *

2b. Illustrate how different problems are solved by addition, subtraction, multiplication and division and be able to explain how the operation used is connected to the solving of the problem. *

2c. Recognize that addition, subtraction, multiplication, and division problem types and associated meanings for the operations (e.g., CCSS, pp. 88–89) extend from whole numbers to fractions and decimals. *

2d. Employ teaching/learning paths for single-digit addition and associated subtraction and single-digit multiplication and associated division, including the use of properties of operations (i.e., the field axioms). *

2e. Compare and contrast standard algorithms for operations on multi-digit whole numbers that rely on the use of place-value units (e.g., ones, tens, hundreds, etc.) with mental math methods students generate. *

2f. Use math drawings and manipulative materials to reveal, discuss, and explain the rationale behind computation methods. *

2g. Extend algorithms and mental math methods to decimal arithmetic. *

2h. Use different representations of the same fraction (e.g., area models, tape diagrams) to explain procedures for adding, subtracting, multiplying, and dividing fractions. (This includes connections to grades 6–8 mathematics.). *

2i. Explain the connection between fractions and division, $a/b = a \div b$, and how fractions, ratios, and rates are connected via unit rates. (This includes connections to grades 6–8 mathematics. See the Ratios and Proportional Relationships Progression for a discussion of unit rate.). *

2j. Explain why the extensions of the operations to signed numbers make sense. *

3. Algebraic Thinking

The successful Mathematics in Elementary Education student can:

3a. Model and communicate their reasoning about quantities and the relationships between quantities using a variety of representations. *

3b. Discuss the foundations of algebra in elementary mathematics, including understanding the equal sign as meaning “is the same [amount] as” rather than a “calculate the answer” symbol. *

3c. Look for regularity in repeated reasoning, describe the regularity in words, and represent it using diagrams and symbols and communicate the connections among these. *

3d. Articulate, justify, identify, and use properties of operations. *

3e. Describe numerical and algebraic expressions in words, parsing them into their component parts, and interpreting the components in terms of a context. *

3f. Use a variety of methods (such as guess and check, pan balances, strip diagrams, and properties of operations) to solve equations that arise in “real-world” contexts. *

4. **Number Theory**
   The successful Mathematics in Elementary Education student can:

   4a. Demonstrate knowledge of prime and composite numbers, divisibility rules, least common multiple, greatest common factor, and the uniqueness (up to order) of prime factorization. *

   4b. Discuss decimal representation and recognize that there are numbers beyond integers and rational numbers. *

TMM022: Mathematics in Elementary Education II *(updated May 8, 2020)*

Typical Range: 4-5 Semester

Hours Recommendation:

It is essential for all teachers of mathematics to understand the reasoning underlying the mathematics they are teaching. They need to understand why various procedures work, how each idea they will be teaching connects with other important ideas in mathematics, and how these ideas develop and become more sophisticated. Furthermore, knowing only the mathematics of the elementary grades is not sufficient to be an effective teacher of elementary grades mathematics. Neither is it sufficient to require that future teachers simply “take more math courses.” This document describes the kinds of mathematics and mathematical experiences that we believe are essential for their mathematical learning and professional development. Exploration in middle grade topics are introduced at the elementary level to begin planting the seed in preparation for advanced teachings.

We take the view that mathematics courses for future teachers must prepare them to do a different kind of work in their mathematics teaching than they likely experienced in their own schooling. With this in mind, we encourage oral and written communication in these classes as both a learning tool and as preparation for handling mathematical questions which arise in their classrooms. For example, we aim for discussion that focuses on the deep mathematical reasoning underlying the computational procedures that are usually taught in elementary school. We recommend this be done by exploring common misconceptions with preservice teachers and by engaging future teachers in activities that require them to interpret their own multiple ways of addressing questions and interpreting children’s work which might be incorrect, incomplete, or different from their adult ways of thinking. We aim to encourage a serious approach to the mathematics through questions (by instructors *and* preservice teachers) about *why* we do things the way we do, what the operations *mean*, what the *units* are on the answers, and how the mathematical ideas of the day *connect* to other mathematical ideas (looking for the “big ideas” in a problem).

We recommend these courses be activity based so that opportunities for deep, connected learning arise while misconceptions are addressed. This requires “good” problems and “good” questioning by instructors. A good problem needs to engage the preservice teachers at an appropriate level of challenge (hard enough that the preservice teacher cannot answer on autopilot). Often this is accomplished by confronting them with misconceptions framed as “a child said this…” and directions to analyze and/or justify the result. The justifications can themselves become a source of deep discussion - one preservice teacher may not understand another’s solution or explanation, a preservice teacher’s correct answer may have a flawed explanation, or a new method may be generated once an array of other methods has been shared. Sometimes the discussion is generated when an instructor says, “I want to list all of the different answers we got before we discuss the reasoning” (which means the class can actually discuss their own wrong answers). All of the above takes a lot of time, so we recommend 8 to 10 credit hours of such work in teacher education programs or prerequisites.

At the same time, we strongly urge that instructors be aware that these are college level mathematics courses. “A credit-bearing, college-level course in Mathematics must use the standards required for high school graduation by the State of Ohio as a basis and must do at least one of the following: 1) broaden, or 2) deepen, or 3) extend the student’s learning.” We recommend dedicated coursework for this content because there is much deep mathematics to be explored in understanding what we loosely term “elementary school mathematics.” This is a mathematical content course. The learning outcomes are all focused on using, justifying, and connecting mathematical concepts and do not address “how to teach.” It is sometimes appropriate to discuss topics which are more directly relevant to a methods course when they serve the purpose of motivating a mathematical discussion, but students should not be assessed on methods in these courses.

The courses should integrate reasoning, flexibility, multiple explanations, and number sense. Leading questions help students to make connections among topics and to develop their own questioning skills (e.g. Have we seen this idea before? How are these two different solution methods related to each other? What does it mean? How do you know? Could you draw a picture to show it? Where did they go wrong? Could we use their idea in this other problem?) Students should understand that mathematics is correct if it makes personal sense and if one can explain it in a way to make sense to others, not if an authority certifies it. Preservice teachers should leave these courses knowing that math makes sense and armed with the underlying knowledge they need to make math make sense for their future students. Elementary students will often come up with their own creative approaches to problems, so future teachers must be able to evaluate their mathematical viability *before* deciding how to respond instructionally. Squashing a child’s idea can be quite harmful. And often children’s ideas are right or almost right.

The learning outcomes that follow are all viewed as essential by the committee (marked with an asterisk) and demonstrate the level of student engagement motivating this course. In addition, learning outcomes are not specific items/topics but rather learning outcomes of course entirety. Institutional courses should provide an integrated experience with learning outcomes woven together throughout the course.

1. **Ratios, Proportional Relationships, and Functions** (this has connections to Grades 6-8) The successful Mathematics in Elementary Education student can:

   1a. Reason about how quantities vary together in a proportional relationship, using tables, double number lines, and tape diagrams as supports. *

   1b. Distinguish proportional relationships from other relationships, such as additive relationships and inversely proportional relationships. *

   1c. Use unit rates to solve problems and to formulate equations for proportional relationships (see measurement). *

   1d. Recognize that unit rates make connections with prior learning by connecting ratios to fractions. *

   1e. View the concept of proportional relationship as an intellectual precursor and key example of a linear relationship. *

1f. Examine and reason about functional relationships represented using tables, graphs, equations, and descriptions of functions in words. In particular, students can examine the way two quantities change together using a table, graph, and equation. *

1g. Examine the patterns of change in proportional and linear relationships and the types of real-world situations these functions can model and contrast with nonlinear relationships. *

2. **Measurement**
   The successful Mathematics in Elementary Education student can:

2a. Explain the general principles of measurement, the process of iterations, and the central role of units (including nonstandard, U.S. customary, and metric units). *

2b. Explain how the number line connects measurement with number through length. *

2c. Understand and distinguish area and volume, giving rationales for area and volume formulas that can be obtained by finitely many compositions and decompositions of unit squares or unit cubes, including but not limited to formulas for the areas of rectangles, triangles, and parallelograms, and volumes of arbitrary right prisms. (This includes connections to grades 6–8 geometry, see the Geometric Measurement Progression.). *

2d. Describe how length, area, and volume of figures change under scaling, focusing on areas of parallelograms and triangles, with counting-number scale factors. *

2e. Informally develop the formulas for area and circumference of a circle and use them in solving real-world problems. *

2f. Attend to precision in measurement with rounding guided by the context. *

2g. Convert between different units both by reasoning about the meaning of multiplication and division and through dimensional analysis. *

3. **Geometry**
   The successful Mathematics in Elementary Education student can:

3a. Understand geometric concepts of angle, parallel, and perpendicular, and use them in describing and defining shapes. *

3b. Describe and reason about spatial locations (including the coordinate plane). *

3c. Informally prove and explain theorems about angles and solve problems about angle relationships. *

3d. Classify shapes into categories and reason to explain relationships among the categories. *

3e. Explain when and why the Pythagorean Theorem is valid and use the Pythagorean Theorem in a variety of contexts. *

3f. Examine, predict, and identify translations, rotations, reflections, and dilations, and combinations of these. *

3g. Understand congruence in terms of translations, rotations, and reflections; and similarity in terms of translations, rotations, reflections, and dilations and solve problems involving congruence and similarity. *

3h. Understand symmetry as transformations that map a figure onto itself. *

4. Statistics and Probability

The successful Mathematics in Elementary Education student can:

4a. Recognize and formulate a statistical question as one that anticipates variability and can be answered with data. *

4b. Understand various ways to summarize, describe, and compare distributions of numerical data in terms of shape, center (e.g., mean, median), and spread (e.g., range, interquartile range). *

4c. Use measures and data displays to ask and answer questions about data and to compare data sets. (This includes connections to grades 6–8 statistics.). *

4d. Distinguish categorical from numerical data and select appropriate data displays. *

4e. Use reasoning about proportional relationships to argue informally from a sample to a population. *

4f. Calculate theoretical and experimental probabilities of simple and compound events, and understand why their values may differ for a given event in a particular experimental situation. *

4g. Explore relationships between two variables by studying patterns in bivariate data. *

TMM023- DISCRETE MATHEMATICS
(March 2021)

Typical Range: 3-4 Semester Hours

A course in Discrete Mathematics specializes in the application of mathematics for students interested in information technology, computer science, and related fields. This college-level mathematics course introduces students to the logic and mathematical structures required in these fields of interest.

TMM023 Discrete Mathematics introduces mathematical reasoning and several topics from discrete mathematics that underlie, inform, or elucidate the development, study, and practice of related fields. Topics include logic, proof techniques, set theory, functions and relations, counting and probability, elementary number theory, graphs and tree theory, base-*n* arithmetic, and Boolean algebra.

To qualify for TMM023 (Discrete Mathematics), a course must achieve all the following essential learning outcomes listed in this document (marked with an asterisk). In order to provide flexibility, institutions should also include the non-essential outcomes that are most appropriate for their course. It is up to individual institutions to determine if further adaptation of additional course learning outcomes beyond the ones in this document are necessary to support their students' needs. In addition, individual institutions will determine their own level of student engagement and manner of implementation. These guidelines simply seek to foster thinking in this direction.

1. **Formal Logic** – Successful discrete mathematics students are able to construct and analyze arguments with logical precision. These students are able to apply rules from logical reasoning both symbolically and in the context of everyday language and are able to translate between them.

   The successful Discrete Mathematics student can:

   1.1. Propositional Logic:

   1.1a. Translate English sentences into propositional logic notation and vice-versa. *

   Sample Tasks:

- Determine if a sentence is a proposition.
- Construct the negation of a statement, the conjunction of two statements, and the disjunction of two statements.
- Make use of the symbols $\neg$, $\rightarrow$, $\vee$, $\wedge$, etc.
- Translate English sentences into propositional logic using appropriate notation, such as "John is healthy and wealthy but not wise".

1.1b. Construct truth tables for statements involving the following logical connectives: negation, conjunction, disjunction, conditional, and biconditional.*

Sample Tasks:

- Construct the truth tables for a given compound proposition.
- Determine if two compound statements are logically equivalent using truth tables.

1.1c. Apply De Morgan's Laws to find negations of statements. *

Sample Tasks:

- Negate an *and* statement.
- Negate an *or* statement.
- Use De Morgan's Laws to negate statements involving intervals such as $2 < x < 7$.

1.1d. Define and use these terms: conditional statement, converse, inverse, contrapositive, biconditional, necessary and sufficient conditions, tautology, contingency, and contradiction.*

Sample Tasks:

- Find the negation, converse, inverse and contrapositive of a given conditional statement.
- Interpret and translate English sentences that express necessary and sufficient conditions into formal logic, such as , "Catching the 8:05 bus is a sufficient condition for me being on time for work." or "A necessary condition for this computer program to be correct is that it not produce error messages during translation."
- Determine if a given compound proposition is a tautology, contradiction, or contingency, such as $p \vee \neg p, p \wedge \neg p$.

1.1e. Apply standard logical equivalences to simplify propositions, and be able to prove that two logical expressions are or are not logically equivalent. *

Sample Tasks:

- Use basic laws of equivalence, such as Commutative laws, Associative laws, Distributive laws, De Morgan's laws, Identity laws, Double Negation law, Idempotent laws, Universal bound laws, and Absorption laws, to simplify propositions.

1.1f. Determine if a logical argument is valid or invalid. Apply standard rules of inference including Modus Ponens, Modus Tollens, Generalization, Specialization, Conjunction, Transitivity, and Elimination. Recognize fallacies such as the Converse Error and the Inverse Error. *

Sample Tasks:

- Determine whether a given argument form is valid or invalid by constructing a truth table.
- Determine whether a given argument form is valid or invalid by applying standard rules of inference.
- Identify errors in reasoning that result in an invalid argument (fallacies), such as the converse error or inverse error.

1.2. Predicates and Quantifiers:

1.2a. Translate between English sentences and symbols for universally and existentially quantified statements, including statements with multiple quantifiers. *

Sample Tasks:

- Translate a quantified statement into a logical expression, such as: "There exists an even prime number.", "Every basketball player is tall.", "For every positive rational number *a*, there exists a positive rational number *b* such that *ab=1.*"
- Make use of the symbols $\forall$ and $\exists$ when translating sentences from English into formal logic and vice-versa.

1.2b. Write the negation of a quantified statement involving either one or two quantifiers. *

Sample Tasks:

- Negate a universal statement, existential statement, and a statement involving two or more quantifiers.

1.2c. Determine if a quantified statement involving either one or two quantifiers is true or false.*

Sample Tasks:

- Determine if a given quantified statement, such as a quantified statement involving real numbers or integers and their properties, is true or false and justify the answer.

2. **Proof Techniques** - Successful discrete mathematics students understand how the rules of inference are used in standard proof techniques such as direct proofs, indirect proofs, proof by cases, and the use of counterexamples to disprove statements. They are able to apply these proof techniques to prove results from elementary number theory and other areas of mathematics.

The successful Discrete Mathematics student can:

2a. Use the direct proof method to prove propositions.*

Sample Tasks:

- Use the direct method to prove propositions about integers, such as *The product of an even integer and an odd integer is even.*
- Use the direct method to prove propositions about rational numbers, such as *The sum of two rational numbers is rational*.
- Use the direct method to prove propositions, such as *If x and y are positive real numbers, then* $\frac{x+y}{2} \geq \sqrt{xy}$.

2b. Identify logical errors and disprove statements. *

Sample Tasks:

- Explain the logical error(s) in a given incorrect "proof".
- Provide counterexamples to disprove statements, such as *All real numbers y can be expressed as* $y = \frac{1}{x}$ *for some real number x*.

2c. Use mathematical induction to prove propositions. *

Sample Tasks:

- Let F_n be the n^{th} term of the Fibonacci sequence. Prove by induction that $F_1^2 + F_2^2 + \ldots + F_n^2 = F_n F_{n+1}$.
- Prove by induction that the sum of the first n positive integers is given by $\frac{n(n+1)}{2}$.

2d. Utilize other proof methods to prove propositions.

Sample Tasks:

- Use a proof by contradiction to prove propositions, such as $\sqrt{2}$ *is irrational.*

- Use a proof by contraposition to prove propositions, such as *If the product of two integers ab is even, then a is even or b is even*.
- Use a proof by cases to justify propositions, such as the *Triangle Inequality*.
- Write a constructive existence proof, such as *There exists a positive integer greater than 2 that can be written both as the sum of two cubes of positive integers and the sum of two squares of positive integers*.
- Write a nonconstructive existence proof, such as *There exist irrational numbers x and y such that* x^y *is rational.*
- Identify when a proposition is vacuously or trivially true.

3. **Set Theory** – Successful Discrete Mathematics students demonstrate an understanding of sets, set operations, set identities, and the cardinality of sets. They can use set notations including those for subsets, unions, intersections, differences, symmetric differences, complements, Cartesian products, the empty set, and power sets. They make use of Venn diagrams as appropriate, can formally verify set identities, and can apply the inclusion-exclusion principle to solve problems. They understand and use the terms cardinality, finite, countably infinite, and uncountably infinite, and can determine which of these characteristics is associated with a given set.

   The successful Discrete Mathematics student can:

   3a. Find subsets, unions, intersections, differences, symmetric differences, complements, power sets, and cross products of sets and use them to solve applied problems. *

   Sample tasks:

   - Determine if one set is a subset of another set.
   - Find the union, intersection, and various cross products of two or more given sets, and use the idea of an empty set as appropriate.
   - Find the difference of two given sets and the symmetric difference of those sets.
   - Determine the complement of a given set from a specified universal set.
   - Find the power set of a given set and determine the number of elements in the power set.

   3b. Use Venn diagrams to solve problems, illustrate set identities, and apply the inclusion-exclusion principle. *

   Sample tasks:

- Use Venn diagrams to demonstrate the ideas of subsets, unions, intersections, differences, symmetric differences, and complements of sets.
- Use Venn diagrams to illustrate various set identities, such as Domination Laws, Idempotent Laws, DeMorgan's Laws, and Absorption Laws.
- Use Venn diagrams to illustrate the principle of inclusion-exclusion for two or three sets.
- Apply the principle of inclusion-exclusion to determine the number of integers in a range of integers that are not divisible by 2 or 3.

4. **Relations** – Successful Discrete Mathematics students demonstrate an understanding of relations and are able to determine their properties. They can identify basic properties of relations, including the reflexive, symmetric, antisymmetric, and transitive properties. They can identify equivalence relations, equivalence classes, and partitions.

   The successful Discrete Mathematics student can:

   4a. Identify basic properties of relations, including reflexive, symmetric, antisymmetric, and transitive properties. Identify equivalence relations, equivalence classes, and partitions.

   Sample tasks:

   - Given the relation $R = \{(a, b) : a \leq b\}$, determine if the relation is reflexive, symmetric, antisymmetric, or transitive.
   - Determine if the relation R on students in a class where aRb if and only if a and b share a birthday (month and day) is an equivalence relation.
   - Show that the relation R on the set of all bit strings of length 5 where aRb if and only if they have the same bits in the first 3 positions is an equivalence relation; then determine the equivalence classes if so.
   - Determine if the relation R on the set of bit strings of length 4 where aRb if and only if they have exactly two bits in common is an equivalence relation. If it is, list the elements in the equivalence class [0110]; if it isn't, explain why.

   4b. Find the reflexive closure, symmetric closure, and transitive closure of a relation on a set.

   Sample tasks:

   - If R is the relation aRb if and only if a divides b on the set of integers, find the symmetric closure of R.
   - Find the smallest relation containing the relation {(1, 3), (2, 3), (4, 1), (4, 4)} that is reflexive and transitive.

5. **Functions-** Successful Discrete Mathematics students demonstrate an understanding of functions and are able to determine their properties. They can determine the domain,

codomain, and range of discrete functions. Given a function and a pre-image, they can determine its image, and given an image they can determine its pre-image(s). They can graph functions, perform composition of functions, find and/or graph the inverse of a function, and use the properties of functions to solve applied problems. They understand the notions of injections, surjections, and bijections, and can determine which of these characteristics are associated with a given function. They can use the notion of a bijection to prove that two sets have the same cardinality. They can analyze the growth of elementary functions and determine their Big-*O* value. They can analyze simple algorithms and compare two algorithms based on computational complexity.

The successful Discrete Mathematics student can:

5a. Determine the domain, codomain, and range of discrete functions. Identify injections, surjections, and bijections, and determine which of these characteristics is associated with a given function. *

Sample tasks:

- Let f be the function from $\mathbb{Z}$ to $\mathbb{Z}$ given by $f(n) = n^2$. Determine the domain, codomain, and range of this function.
- Let f be the function from $\mathbb{Z}$ to $\mathbb{Z}$ given by $f(n) = n^2$. Determine whether this function is one-to-one and/or onto.
- Let f be the function from ($\mathbb{Z}$ x $\mathbb{Z}$) to $\mathbb{Z}$ given by $f(m, n) = m + n$. Determine whether this function is one-to-one and/or onto.
- Let f be the function from $\mathbb{Z}$ to $\mathbb{Z}$ given by $f(n)$ = Floor(1.5*n*). Determine whether this function is one-to-one and/or onto.
- Find a bijection from the even integers to the integers that are a multiple of 5 that proves these two sets have the same cardinality.

5b. Demonstrate an understanding of the terms cardinality, finite, infinite, and uncountably infinite, and determine the cardinality of a given set. Use the notion of a bijection to prove that two sets have the same cardinality.

Sample tasks:

- Determine whether the set of odd negative integers is finite, countably infinite, or uncountably infinite. If it is countably infinite, exhibit a one-to-one correspondence between the set of positive integers and the set of odd negative integers.
- Determine whether the set of real numbers between 0 and 3 is finite, countably infinite, or uncountably infinite. If it is countably infinite, exhibit a one-to-one correspondence between the set of positive integers and this set.

- Give an example of two uncountably infinite sets such that their intersection is countably infinite.

5c. Determine the Big-*O* estimate for basic functions.

Sample tasks:

- Show that $f(x) = x^3 + 7x^2 + 5x + 3$ is $O(x^3)$.
- Show that $g(x) = x^3$ is not $O(x^2)$.
- Determine an estimate for the big-*O* value of the sum of the first n positive integers.

6. **Sequences**- Successful Discrete Mathematics students demonstrate an understanding of sequence as a function whose domain is a subset of the integers. They can identify special sequences such as Fibonacci, factorial, arithmetic, and geometric sequences and can apply recursion.

   The successful Discrete Mathematics student can:

6a. Define a sequence as a function whose domain is a subset of the integers. Identify arithmetic and geometric sequences, and the factorial sequence. *

Sample tasks:

- Find the first five terms of the sequence $\{a_n\}$, $n \geq 0$, where $a_n = 3(-2)^n + 5$.
- List the first ten terms of the arithmetic sequence whose first term is 5 and whose fourth term is 14.
- List the first 5 terms of the geometric sequence whose first term is 4! and whose third term is 96.

6b. Describe how sequences and sets can be defined recursively. Identify the Fibonacci sequence. *

Sample tasks:

- Give a recursive definition for the factorial sequence.
- Give a recursive definition for the Fibonacci sequence.
- Given $f(0) = 3$ and $f(n + 1) = 2^{f(n)}$, find $f(1)$ and $f(2)$.
- Given $f(1) = 5$, $f(2) = 2$, and $f(n+2) = 2f(n + 1) - f(n)$, find $f(3)$, $f(4)$, and $f(5)$.
- Let S be the set defined recursively by $3 \in S$, and if $x \in S$ and $y \in S$ then $x + y \in S$. Describe, in words, the set S.

7. Counting and Probability - A successful student will demonstrate understanding of the fundamental principles of counting and probability and the ability to apply them in a wide variety of situations.

The successful Discrete Mathematics student can:

7a. Solve counting problems involving the multiplication rule and permutations/combinations. (with and without repetition). *

Sample tasks:

- Use the multiplication rule to calculate the number of ways a process can be performed. For instance, suppose a computer installation requires one of four input/output units (A, B, C, and D) and one of three central processing units (X, Y, and Z). Determine how many different installations are possible.
- Compute the number of permutations of a set of *n* elements, for instance, the number of different ways you can arrange the letters in the word COMPUTER.
- Calculate the number of *r*-permutations of a set of *n* elements given by the formula $P(n,r) = \frac{n!}{(n-r)!}$.
- Use the binomial coefficient $C(n,r) = \binom{n}{r}$ to count the number of subsets of size *r* that are in a set of size *n.* For instance, find the number of five-member groups chosen from a set of twelve people.

7b. Apply the Addition Rule and the Principle of Inclusion and Exclusion. *

Sample tasks:

- Calculate the number of elements in a union of mutually disjoint finite sets.
- Calculate the number of elements in a union of sets when some of the sets have nonempty intersection.

7c. Apply basic principles of discrete probability.

Sample tasks:

- Calculate the probability of the union of events.
- Calculate the probability of the intersection of independent events, conditional probability, and probability of the intersection of dependent events.

7d. Investigate Pascal's Triangle and/or apply the Binominal Theorem.

Sample tasks:

- Given the n^{th} row of Pascal's Triangle students calculate the $(n + 1)^{th}$ row.
- Identify the relation between the binomial coefficients and the Pascal's Triangle entries.
- Use the binomial theorem to expand expressions such as $(1 + x)^6$.

7e. Apply the Pigeonhole Principle.

Sample tasks:

- Explain the pigeonhole principle.
- Apply the pigeonhole principle to problems, for instance, showing that any set of six distinct positive integers less than 11 contains a pair whose sum is 11.

8. Elementary Number Theory

The successful Discrete Mathematics student can:

8a. Determine if a proposed statement involving concepts from elementary number theory is true or false.

Sample tasks:

- Construct proofs involving concepts from elementary number theory such as properties of even and odd integers and divisibility.
- Determine if statements are true or false. Justify answers with a proof or a counterexample, as appropriate.

8b. State and use the Division Algorithm.

Sample tasks:

- State the Division Algorithm.
- Use the Division Algorithm to determine the equivalence relations in modular arithmetic.

8c. Apply modular Arithmetic.

Sample tasks:

- Perform additions, subtractions, and multiplications modulo n.

8d. State and use the Fundamental Theorem of Arithmetic.

Sample tasks:

- State the Fundamental Theorem of Arithmetic.
- Find the unique factorization of a given integer.
- Use the Fundamental Theorem of Arithmetic as a tool to find solutions to Diophantine equations.

8e. State and use the Euclidean Algorithm.

Sample tasks:

- State the Euclidean Algorithm.
- Use the Euclidean algorithm to hand-calculate the greatest common divisor of a pair of integers.

9. Graphs and Trees Theory - Successful Discrete Mathematics students understand the basic terminology used in graph theory and are able to use graphs and trees to model real-world situations. They are able to determine whether or not a given graph contains an Euler circuit/path and/or a Hamilton circuit/path, and construct those circuit(s)/path(s) if they exist. They are able to determine whether or not a graph is planar and apply Euler's formula if so. They understand the basic properties of *n*-ary trees, how to create decision trees, how to perform traversals, and use them to solve problems.

The successful Discrete Mathematics student can:

9a. Identify basic features of graphs, construct graphs with given properties, and represent graphs using matrices or lists.

Sample Tasks:

- Identify the neighbors of and the degree of each vertex in a graph.
- Identify the bridges in a given graph.
- Explain whether a given sequence of vertices determines a path in a graph, and if so, whether or not the path is simple and/or a circuit.
- Determine whether a given graph is simple or a multigraph, directed or undirected, connected or disconnected, and whether or not it is bipartite.
- Use a graph to model a real-world situation and explain how particular graph features align with the real-world situation.
- Create the complete graph K_n and the complete bipartite graph $K_{n,m}$ for given positive integers n, m.
- Represent a given graph with an adjacency matrix or list.

- Construct a graph represented by a given adjacency matrix or list.

9b. Determine whether or not a given graph has an Euler circuit, Euler path, Hamilton circuit, and/or Hamilton path and construct them if so.

Sample Tasks:

- Use Fleury's algorithm to find an Euler circuit/path in a given graph if one exists or explain why the graph does not have an Euler circuit/path.
- Determine the values of *n, m* for which K_n and $K_{n,m}$ have an Euler circuit or path.
- Identify a Hamilton circuit/path in a given graph if one exists or explain why the graph does not have a Hamilton circuit/path.

9c. Determine whether or not a given graph is planar and apply Euler's formula to planar graphs.

Sample Tasks:

- Determine whether or not a given graph is planar and draw a representation where no edges cross if so.
- Apply Euler's formula to make determinations about planar graphs with given characteristics.

9d. Identify trees and *n*-ary trees, create *n*-ary trees for specified applications including decision trees, and perform tree traversals.

Sample Tasks:

- Determine if a given graph is a tree, and if so, whether or not it is a *n*-ary tree.
- Create a decision tree to represent an algorithm, such as a sorting algorithm.
- Explain the process that is represented by a given decision tree.
- Construct a binary search tree for a list of strings.
- Perform tree traversals using preorder, inorder, and postorder traversal algorithms.
- Explain how traversals can be used to solve application problems.

10. Base-*n* Arithmetic- Successful discrete mathematics students understand base-*n* systems with a focus on base-2 (binary), base-8 (octal), and base-16 (hexadecimal) systems.

The successful Discrete Mathematics student can:

10a. Perform arithmetic in various base-*n* systems.

Sample Tasks:

- Count in binary, octal, and hexadecimal.
- Add, subtract, and multiply in binary, octal, and hexadecimal systems.

10b. Convert between various base-*n* systems.

Sample Tasks:

- Convert numbers between decimal, binary, octal, and hexadecimal systems.

10c. Represent signed binary numbers with 1 and 2's complements.

Sample Tasks:

- Represent signed binary numbers with 1 and 2's complements.
- Use 1 and 2's complements to add and subtract signed binary numbers.

11. Boolean Algebra/Logic - Successful discrete mathematics students understand the Boolean algebra structure and how it is related to logic networks. They are able to minimize Boolean algebra expressions and logic networks.

The successful Discrete Mathematics student can:

11a. Prove and/or apply properties of the Boolean algebra structure.

Sample Tasks:

- Determine whether a given mathematical structure is a Boolean algebra.
- Prove various properties of Boolean algebras.
- Construct a logic network to represent a Boolean expression.
- Write the truth function for a Boolean expression or logic network.
- Define a Boolean function and given a Boolean function, describe it using an input/output table.
- Write Boolean expressions in the canonical sum-of-products form given truth functions.

11b. Minimize Boolean algebra expressions and logic networks.

Sample Tasks:

- Minimize Boolean expressions and logic networks using Karnaugh maps and/or the Quine-McCluskey procedure.

TMM024- Life Science Calculus I *(Updated 8 February 2021)*

Suggested Number of Credit Hours: 4

This is the first course in a two-semester sequence of calculus courses intended for students majoring in the biological or environmental sciences and/or preparing for admission to medical, pharmaceutical, dental, veterinary, or other life-science-related professional schools. Students in this sequence must reason with limits, derivatives, integrals, and differential equations to describe and gain insight into biological processes and populations. Algebraic, logarithmic, exponential, and trigonometric functions are all used to model concepts from the life sciences. Questions from the life sciences should be used to motivate the content of the course, and the concepts and techniques taught should be used explicitly to answer those questions.

To qualify for TMM024 (Life Science Calculus I - LSCI), a course must achieve all the following essential learning outcomes listed in this document (marked with an asterisk). These make up the bulk of a Life Sciences Calculus I course. Courses that contain only the essential learning outcomes are acceptable from the TMM024 review and approval standpoint. It is up to individual institutions to determine further adaptation of additional course learning outcomes of their choice to support their students' needs. In addition, individual institutions will determine their own level of student engagement and manner of implementation. These guidelines simply seek to foster thinking in this direction. Sample tasks are listed to help clarify the intention of the essential learning outcomes, but no specific sample task is required for approval. Institutions are encouraged to tailor specific applications to the population of life science students at their institution.

In a Life Science Calculus I (TMM024) course, students should:

- develop effective thinking and communication skills;
- operate at a high level of detail;
- state problems carefully, articulate assumptions, understand the importance of precise definition, and reason logically to conclusions;
- identify and model essential features of a complex situation, modify models as necessary for tractability, and draw useful conclusions;
- deduce general principles from particular instances;
- use and compare analytical, visual, and numerical perspectives in exploring mathematics;
- assess the correctness of solutions, create and explore examples, carry out mathematical experiments, and devise and test conjectures;

- recognize and make mathematically rigorous arguments;
- read mathematics with understanding;
- communicate mathematical ideas clearly and coherently both verbally and in writing to audiences of varying mathematical sophistication;
- approach mathematical problems with curiosity and creativity, persist in the face of difficulties, and work creatively and self-sufficiently with mathematics;
- learn to link applications and theory;
- learn to use technological tools; and
- develop mathematical independence and experience open-ended inquiry.

– Adapted from the MAA/CUPM 2015 Curriculum Guide

1. **Modeling with Elementary Functions:** Successful LSCI students have a deeper understanding of elementary functions beyond what is learned in prerequisite courses. They will communicate and interpret information about functions given algebraically, graphically, numerically, and verbally. Students will appreciate linear, exponential, logarithmic, and logistic functions as tools to model phenomena in applications. A key concept is that linear functions are those functions with constant rates of change, and that exponential growth and decay is an appropriate model when a constant percentage rate of change occurs. (Trigonometric functions are contained in Life Science Calculus II).

   The successful LSCI student can:

   1.1. Construct a linear model from given linear data. Interpret the slope of a linear model as a constant rate of change. Recognize a situation with constant rate of change as appropriate for a linear model.*

   1.2. Use exponential functions to model growth and decay with constant percentage change. Compute the half-life or doubling time of an exponentially-modelled quantity. Examples include population growth and drug concentration. *

   1.3. Use logistic functions to model natural phenomena such as bounded population growth. Identify the horizontal asymptotes and point of steepest increase/decrease. *

   1.4. Interpret and generate logarithmic scale graphs. Use logarithmic scale graphs to distinguish different types of growth. *

2. **Limits and Continuity:** Successful Calculus students demonstrate understanding of the concepts of limit and continuity whether described verbally, numerically, graphically, or algebraically (both explicitly and implicitly).

   The successful LSCI student can:

2.1. Evaluate limits using tables of function values. This includes one- and two-sided limits at a point as well as limits at infinity and infinite limits. *

2.2. Evaluate limits using a graph of a function. This includes one- and two-sided limits at a point as well as limits at infinity and infinite limits. *

2.3. Use the limit to describe graphical attributes of a function and interpret the meaning of these attributes in a given life science context. *

Sample Task:

- Use asymptotic functions to model quantities that are approaching some theoretical limit, such as a population carrying capacity or the amount of a drug in the bloodstream.

2.4. Evaluate limits algebraically using limit laws. *

2.5. Determine continuity of a simple function from its graph. *

2.6. Use the definition of continuity to determine whether a given function is continuous. *

3. **Rate of Change:** Successful LSCI students have a robust understanding of the concept of rate of change and how it relates to slope on a graph, and the increase or decrease of a quantity in context. They can explain this concept verbally and graphically as well as recognize that derivatives are rates of change.

The successful LSCI student can:

3.1. Interpret the slope of a secant line as an average rate of change of a quantity using correct units. *

3.2. Relate average rate of change over an interval to instantaneous rate of change at a point. Decide if a given graph shows a tangent line or a secant line. *

3.3. Estimate instantaneous rate of change from a graph using a tangent line. *

3.4. Interpret the slope of a tangent line as instantaneous rate of change of a quantity, using correct units. *

4. **Differentiation:** Successful LSCI students are able to compute derivatives and interpret them as rates of change. They can use the formal limit definition of derivative to compute simple derivatives and explain where this limit definition comes from. They can also use product, quotient, and chain rules to compute more complicated derivatives and decide when to use these

rules when given a contextual rate-of-change problem.

The successful LSCI student can:

4.1. Find the derivative of a function using the limit definition of derivative. This computation can be performed numerically (with tables) and algebraically. *

4.2. Compute derivatives of elementary algebraic and transcendental functions using the power rule, product rule, quotient rule, and chain rule. *

4.3. Interpret the value of a derivative as a rate of change with units. *

4.4. Determine where a derivative does not exist, using a function's formula or its graph. *

4.5. Compute higher-order derivatives e.g. second derivatives. *

5. **Applications of Derivatives:** Successful LSCI students can use derivatives to solve optimization problems, to describe graphs, and to explain natural phenomena (such as velocity and acceleration).

The successful LSCI student can:

5.1. Use the first and second derivative of a function to gather information about the function, including: intervals of increase/decrease, critical points, relative and absolute extrema, concavity, and inflection points. *

Sample Task:

- Find the inflection point of a given breast cancer tumor growth function and describe its significance.

5.2. Solve optimization problems in the life science context using first and second derivatives. *

Sample Task:

- Find the maximum and minimum of the therapeutic window of a drug

6. **Integration:** Successful LSCI students are familiar with definite and indefinite integrals. Deeper understanding of integration may be gained in Life Sciences Calculus II.

The successful LSCI student can:

6.1. Use Riemann sums to estimate definite integrals. *
6.2. Evaluate definite integrals using the Fundamental Theorem of Calculus. *
6.3. Evaluate indefinite integrals using basic antiderivative formulas e.g. power rule. *
6.4. Answer questions motivated by the life sciences by computing areas under graphs. *

Sample Task:

- Analyze pathogenesis curves of diseases.

6.5. Interpret the meaning of a definite integral of a function in terms of areas of regions between the graph of the function and the x-axis. *

Sample Tasks:

- The student explains the conditions under which the definite integral of a function gives the area between that function and the x-axis, and the precautions that must be taken when using definite integrals to calculate area.
- Given the graph of a function and the area of each region enclosed by the graph and the x-axis, the student finds the value of a given definite integral of that function.

6.6. Find indefinite and definite integrals using the method of substitution. *

Sample Tasks:

- Given an integral, the student identifies substitution as an appropriate technique.
- The student chooses an optimal expression to convert to a new variable and restates the integral in terms of that new variable.
- The student converts any integration limits to the new variable.
- The student evaluates the transformed integral and recovers the format of the original function.
- The student solves life science application problems that require the method of substitution.

TMM025 – Life Science Calculus II *(Updated 8 February 2021)*

Suggested Number of Credit Hours: 3

This is the second course in a two-semester sequence of calculus courses intended for students majoring in the biological or environmental sciences and/or preparing for admission to medical, pharmaceutical, dental, veterinary, or other life-science-related professional schools. Students in this sequence must reason with limits, derivatives, integrals, and differential equations to describe and gain insight into biological processes and populations. Algebraic, logarithmic, exponential, and trigonometric functions are all used to model concepts from the life sciences. Questions from the life sciences should be used to motivate the content of the course, and the concepts and techniques taught should be used explicitly to answer those questions.

To qualify for TMM025 (Life Science Calculus II - LSCII), a course must achieve all the following essential learning outcomes listed in this document (marked with an asterisk). These make up the bulk of a Life Sciences Calculus II course. Courses that contain only the essential learning outcomes are acceptable from the TMM025 review and approval standpoint. It is up to individual institutions to determine further adaptation of additional course learning outcomes of their choice to support their students' needs. In addition, individual institutions will determine their own level of student engagement and manner of implementation. These guidelines simply seek to foster thinking in this direction. Sample tasks are listed to help clarify the intention of the essential learning outcomes, but no specific sample task is required for approval. Institutions are encouraged to tailor specific applications to the population of life science students at their institution.

In a Life Science Calculus II (TMM025) course, students should:

- develop effective thinking and communication skills;
- operate at a high level of detail;
- state problems carefully, articulate assumptions, understand the importance of precise definition, and reason logically to conclusions;
- identify and model essential features of a complex situation, modify models as necessary for tractability, and draw useful conclusions;
- deduce general principles from particular instances;
- use and compare analytical, visual, and numerical perspectives in exploring mathematics;
- assess the correctness of solutions, create and explore examples, carry out mathematical experiments, and devise and test conjectures;
- recognize and make mathematically rigorous arguments;
- read mathematics with understanding;
- communicate mathematical ideas clearly and coherently both verbally and in writing to audiences of varying mathematical sophistication;
- approach mathematical problems with curiosity and creativity, persist in the face of difficulties, and work creatively and self-sufficiently with mathematics;
- learn to link applications and theory;
- learn to use technological tools; and

- develop mathematical independence and experience open-ended inquiry.

– Adapted from the MAA/CUPM 2015 Curriculum Guide

1. Modeling with trigonometric Functions: Successful LSCII students understand how to model periodic phenomena by the use of trigonometric functions. They expand their knowledge of the concepts and rules of differential and integral calculus to trigonometric functions.

The successful LSCII student can:

1.1. Explain the structure of the graphs of $f(x) = \sin(x)$, $g(x) = \cos(x)$ (amplitude, period, horizontal and vertical intercepts) using the unit circle definition of the functions. *

1.2. Explain the role of the parameters A, B, C and D in the expressions $f(x) = A\sin(Bx + C) + D$ and $g(x) = A\cos(Bx + C) + D$. *

1.3. Recognize periodic phenomena and recognize that such phenomena can be modeled by functions of the form $f(x) = A_1\cos(B_1x + C_1) + A_2\sin(B_2x + C_2)$. *

Sample Tasks:

- The student determines the period and the amplitude of a function from a graph or a data set.
- Given a function which is defined on a bounded interval, the student defines and graphs its periodic extension.
- The student uses technology to explore the roles of the parameters of a function such as $f(x) = A\sin(Bx) + \cos(x)$.

1.4. Use technology to model given periodic data by finding values of the parameters in the expression $f(x) = A_1\cos(B_1x + C_1) + A_2\sin(B_2x + C_2)$. *

2. Calculus of Trigonometric Functions: Successful LSCII students can compute derivatives and integrals of trigonometric functions and interpret the results in context.

The successful LSCII student can:

2.1. Apply the concepts and rules of derivatives to functions to the basic trigonometric functions. *

Sample Tasks:

- The student uses a graph of $f(x) = \sin(x)$ to estimate the slope of a tangent line and approximate the derivative of $f(x)$ at a given point.
- The student computes the derivative of $y = \tan(x)$ using the quotient rule.
- Given a sinusoidal function model of the circadian rhythm for the body temperature of a mammal, the student finds and plots the derivative and interprets the plot.
- The student computes the extrema of a damped oscillation function.

2.2. Apply the concepts of integral calculus to the basic trigonometric functions. *

Sample Task:

- The student integrates y=tan(x) by using substitution.

3. Applications of Definite Integrals: Successful LSCII students can identify a definite integral of a function in terms of areas of regions between the graph of the function and the x-axis and use definite integrals to calculate areas of bounded regions. Students interpret a definite integral as an accumulation of change and apply this understanding to the life science calculus setting.

The successful LSCII student can:

3.1. Calculate area of bounded regions. *

Sample Tasks:

- The student symbolizes measurement of the area between a curve and the x-axis with a single definite integral or a sum or difference of definite integrals.
- The student symbolizes measurement of the area between two curves with a single definite integral or a sum or difference of definite integrals.
- The student calculates area measurement using the Fundamental Theorem of Calculus.

3.2. Interpret a definite integral as an accumulation of change and apply this understanding to the life science calculus setting. *

Sample Tasks:

- When given an applied accumulated change problem with relevance to the life sciences, the student recognizes that rate of change is known and accumulated change is being sought.

- The student symbolizes accumulated change with an appropriate definite integral.
- The student calculates accumulated change using the Fundamental Theorem of Calculus and states the appropriate units.
- The student interprets the meaning of a definite integral equation in the life science context. Example: "What is the meaning of the statement $\int_0^{10} \frac{dh}{dt}\,dt = 6$, if $\frac{dh}{dt}$ represents the growth rate of a tree and $h(t)$ is the height measured in feet and t is the time in years?"
- Using a sinusoidal model for the rate of airflow into the lungs, the student calculates the total amount of air inhaled in one cycle.

4. Integration Techniques: Successful LSCII students extend their knowledge of integration to find indefinite and definite integrals using integration by parts and partial fraction decomposition.

The successful LSCII student can:

4.1. Find indefinite and definite integrals using the method of integration by parts. *

Sample Tasks:

- Given an integral, the student identifies integration by parts as an appropriate technique.
- The student separates the integrand into two factors and correctly assigns each factor to the appropriate part of the integration by parts formula.
- The student applies the integration by parts formula and finds the correct integral.
- The student applies integration by parts on expressions which require multiple iterations of the method of integration by parts.
- The student solves life science application problems that require the method of integration by parts.

4.2. Find indefinite and definite integrals using the method of partial fraction decomposition. *

Sample Tasks:

- Given an integral, the student identifies partial fraction decomposition as an appropriate integration technique.
- The student decomposes a rational function into the sum of a polynomial and partial fractions.
- The student applies known formulas and techniques to integrate the decomposed form of the function.

- The student solves life science application problems that require the method of partial fraction decomposition.

5. Differential Equations: The successful student understands a differential equation as the mathematical model of a situation in which a quantity and its rate of change are dependent on one another. Students are able to use analytical as well as graphical methods. They can describe the behavior of solutions and interpret it in connection with physical situations. They can apply their understanding to a variety of life science problems.

The successful LSCII student can:

5.1. Solve separable differential equations by analytical methods. *
Sample Tasks:

- The student gives examples of differential equations.
- The student decides if a given function is a solution of a differential equation.
- Given the general solution of a differential equation and an initial condition, the student finds the particular solution of the initial value problem.
- The student decides if a given differential equation is separable.
- The student describes the steps of solving a differential equation using separation of variables.
- The student solves a first-order differential equation using separation of variables.
- The student analyzes the end behavior of a solution by computing a limit at infinity.

5.2. Understand the relationship between slope fields and solution curves of first-order differential equations. *
Sample Tasks:

- The student constructs a slope field associated with a first-order differential equation.
- The student matches a slope field to a given first-order differential equation.
- The student uses a slope field and an initial condition to approximate a solution curve of a differential equation.

5.3. Use a differential equation to interpret a physical situation in which a quantity and its rate of change are dependent on one another and apply this understanding to various life science problems. *

Sample Tasks:

- Given a physical situation described in words, the student models it by a differential equation or an initial value problem.
- The student expresses constant relative growth rate of a population by a differential equation.
- The student formulates a differential equation to represent population growth under external influences (such as emigration or harvesting).
- The student models population growth by a logistic equation.
- The student uses a linear differential equation to describe a cooling process.
- The student represents a mixing process (such as a continuous infusion, or pollution of a body of water) by a linear differential equation.
- The student explains the differences between an initial value problem modeling an infusion and an initial value problem modeling a single injection.
- The student critiques the reasonableness of a differential equation model by analyzing the qualitative behavior of the solutions.
- The student discusses the reasonableness of a model by analyzing the end behavior of the solutions.

5.4. Perform a qualitative analysis of an autonomous differential equation, without computing analytical solutions. *

Sample Tasks:

- The student determines the equilibrium solutions of a given autonomous differential equation using analytical methods.
- The student identifies the equilibrium solutions from a graph of $\frac{dy}{dx}$ as a function of y.
- The student analyzes a slope field to identify the equilibrium solutions, and to classify their stability.
- The student constructs a phase line and uses it to determine the stability of the equilibrium solutions.
- The student uses the stability behavior of the equilibrium solutions to sketch typical solution curves.
- The student predicts the end behavior of particular solutions by analyzing the stability of the equilibrium solutions.

Diversity, Equity, and Inclusion (DEI) *(Updated March 30, 2021)*

The Ohio Transfer 36 requires at least 12 semester hours of elective course credit. Diversity, Equity, and Inclusion (DEI) is classified as an Ohio Transfer 36 electives option.

- Learning outcomes 1-4 are considered essential (marked with an asterisks) and required for course approval.
- Institutional course submissions must contain either learning outcomes 5 or 6 (choose at least one) to be considered for course approval.

Learning Outcomes:

1. Describe identity as multifaceted and constituting multiple categories of difference such as race, color, language, religion, national origin, gender, sexual orientation, age, socio-economic status, and intersectionality as operating by individual and group. *
2. Describe how cultures (including their own) are shaped by the intersections of a variety of factors such as race, gender, sexuality, class, disability, ethnicity, nationality, and/or other socially constructed categories of difference. *
3. Recognize the complex elements of cultural biases on a global scale by identifying historic, economic, political, and/or social factors, such as ethnocentrism, colonialism, slavery, democracy, and imperialism. *
4. Recognize how sociocultural status and access to (or distribution of) resources are informed by cultural practices within historical, social, cultural, and economic systems. *

Choose At least One:

5. Articulate the meaning of empathy and its role in strengthening civic responsibilities and reducing the negative impact of societal stereotypes.
6. Demonstrate empathy by successfully interpreting intercultural experiences from one’s own and others’ worldview.