Beyond Lecture

Active-learning strategies in postsecondary mathematics

The purpose of education is learning, not teaching. It seems so obvious.

Yet, even today, “good teaching” continues to draw more attention from those who make and carry out education policy than “effective learning.” And, the academic lecture continues to be the dominant feature of instruction in most college classrooms.

Why? Is it because the standard lecture, or even a great lecture, is the best way to advance learning – to ensure that as many students as possible master the course’s learning outcomes?

The answer is, once again, obvious. No! Research tells us that the passivity of the lecture experience leads to lower levels of engagement – and less learning. It tells us that learning is enhanced when it is experiential and active.

To be sure, there is no single, best pedagogical approach in any classroom. Just as students learn in a multitude of ways, the most effective teachers are those who make use of multiple instructional approaches: problem-solving exercises, brief lectures followed by discussion, small group work, “flipped” classes, simulations, hands-on experimentation, other forms of inquiry-based or active learning, and more.

**What is active learning?**

Active learning engages students in the process of learning through a series of classroom (and outside the classroom) activities and/or discussion. Instead of expecting them to passively listen to an expert and learn, it gives students opportunities to actively engage with content in ways that lead to positive learning outcomes. Arguably, forms of inquiry-based learning are the most well-known examples of active learning in mathematics.

At a March 2017 faculty workshop co-hosted by the Ohio Mathematics Initiative and the Ohio Articulation and Transfer Network, Dr. Carol Schumacher, professor of mathematics and chair of the faculty at Kenyon College, set out some reasons why active learning is often the best way to teach, in part because it shifts students’ attention from receptivity to discovery:

**continued on page 2, Active Learning**
Active learning and the mathematics community

Traditional education with its focus on teaching, not learning, worked for most of today’s postsecondary mathematics faculty. It’s one of the things that attracted them to the teaching profession, and it explains why traditional approaches to instruction are still popular in most mathematics classrooms.

Given that traditional methods of instruction are not meeting the needs of students, why is the mathematics community just now beginning to wake up to the importance of active learning?

Not true, says David Bressoud, a past president of the Mathematical Association of America (MAA). In a recent blog, Dr. Bressoud outlines the MAA’s long-standing promotion — as early as the 1981 report of its Committee on the Undergraduate Program in Mathematics (CUPM) — of active learning. ‘Asserting that the MAA has never ceased in its advocacy for active learning, Dr. Bressoud writes, “It is a cry to which many have responded, but which has recently been rediscovered and promoted with urgency as chairs, deans, provosts, and presidents have come to realize that the way mathematics instruction has traditionally been organized cannot meet our present needs, much less those of the future.”

The committee’s 1981 report urged teachers to guide students to discover new math for themselves, rather than present them with concisely sculptured theories. In 2015, the committee recommended that students develop mathematical independence and experience open-ended inquiry.

Dr. Bressoud suggests that we are fortunate to have this foundation on which to build, now that there is broad recognition of the importance of active-learning strategies in the teaching and learning of mathematics.


continued from page 1, Active Learning

▪ It helps students through their floundering, when they are “stumped.”
▪ It gives students a learner-centered, as opposed to an instructor-centered, environment.
▪ It tells students that they “can do it.”
▪ It takes more time, but can be transformative.

While acknowledging that there are multiple approaches to active learning, Professor Schumacher said all of them share some common features, such as the following:

▪ They put a priority on student-to-student communication.
▪ They place an emphasis on student-centered work.
▪ They require ample student support, including course materials that guide students to make mathematical connections for themselves.
▪ They involve students working collectively.
▪ They turn the instructor into a coach.
▪ They focus on inquiry-based learning.
▪ They produce higher levels of achievement and student satisfaction, if delivered effectively.

Active learning: multiple approaches and a range of possibilities

In a 2015 American Mathematical Society (AMS) blog, Benjamin Braun, et. al., remind us that while it is common to associate inquiry-based learning and active learning with a particular teaching technique that allows students to work independently — or in small groups — in a classroom environment with little or no lecturing by the instructor, these terms represent a full spectrum of teaching styles, techniques, and settings. What is common to these techniques is a belief that students learn best by confronting tasks that challenge them to question their assumptions, work collaboratively with others, and receive coaching and support — as needed — from instructors as they take responsibility for their own learning.¹

As described by Professor Braun and his associates, these learning strategies include the following — beginning with techniques that fall closer to the “all telling” end of the spectrum and ending with techniques closer to the “all student discovery” end:

**Lectures with active learning techniques.** One example of this technique is “think-pair-share,” in which the instructor provides students with a short task — perhaps a short computational problem or a step in a proof to complete. After giving students a brief period of time to independently consider the task, they are asked to compare their answers with the people sitting around them, or with members of their small work team. Finally, some or all students are asked to share their answers in some manner, either with the groups next to them or with the entire class.
“In the context of mathematics, inquiry-based learning approaches engage students in exploring mathematical problems, proposing and testing conjectures, developing proofs or solutions, and explaining their ideas. As students learn new concepts through argumentation, they also come to see mathematics as a creative human endeavor to which they can contribute. Consistent with current socio-constructivist views of learning, IBL methods emphasize individual knowledge construction supported by peer social interactions.”


“A student-centered instructional approach places less emphasis on transmitting factual information from the instructor, and is consistent with the shift in models of learning from information acquisition (mid-1900s) to knowledge construction (late 1900s). This approach includes

- more time spent engaging students in active learning during class;
- frequent formative assessment to provide feedback to students and the instructor on students’ levels of conceptual understanding; and
- in some cases, attention to students’ metacognitive strategies as they strive to master the course material.”


continued from page 2, Active Learning

This process energizes students during lectures and builds their capacity to develop and explain their thinking to peers. The process is relatively simple and is suitable for use in almost any environment, including medium- and large-lecture settings.

**Inverted or “flipped” classes.** In this approach, presentations of basic definitions, examples or proofs are given to students, usually as videos or assigned readings before the class meets. Then class time is used for tasks (e.g., small group discussions, sequenced activity worksheets, or student presentations of their work) with the instructor serving as mediator or coach. The structure of the inverted classroom environment can be used to support in-class tasks with higher levels of cognitive demand and more intensive active learning.

**Mathematics emporium.** As an environment that supports active learning, the typical mathematics emporium is centered around a large room filled with computer workstations, in which students progress through self-paced online courses and work collaboratively with the support of faculty, teaching assistants, or tutors. Many emporium models have no lecture component. And, most mathematics emporiums have been developed to handle remediation issues and foundational courses, such as developmental mathematics and college algebra.

**Laboratory courses.** For the past 25 years, many mathematics courses have been structured around exercises and computer lab activities using programs such as Mathematica, Maple, and MATLAB. With computer technology, there are multiple opportunities for active learning and students are engaged at higher levels of cognitive effort.

Regarding these and other active-learning approaches, Braun, et. al., make it clear that faculty do not need to jump completely to all-discovery if they wish to try inquiry-based learning methods.

**Active learning: Does it really produce better results?**

Active learning takes tremendous patience – a willingness on the instructor’s part to give students enough time to wrap their heads around the ideas. It also requires many learning resources, which can be difficult to produce.

Yet, faculty who use active learning strategies will tell you that the benefits for students are striking: greater knowledge, longer retention, and better understanding.

This perspective is supported by substantial research – most notably a 2014 study in the *Proceedings of the National Academy of Sciences*, which meta-analyzed 225 studies that reported data on student performance and failure rates in undergraduate science, technology, engineering, and mathematics (STEM) courses taught using traditional lecture versus active learning approaches.² According to Scott

continued on page 4, Active Learning
Active learning resources
Bhatia, A. (2014). “Active learning leads to higher grades and fewer failing students in science, math and engineering.” Science (April 12)

continued from page 3, Active Learning
Freeman, the report’s principal author, “The impact of these data should be like the Surgeon General’s report on ‘Smoking and Health’ in 1964; they should put to rest any debate about whether active learning is more effective than lecturing.”

Having conducted the largest and most comprehensive meta-analysis of undergraduate STEM education to date, Professor Freeman and his colleagues concluded:

- **Students in a traditional lecture course are 1.5 times more likely to fail, compared to students in courses with active learning.** The authors found that 34% of students failed their course under traditional lecturing, compared to 22% of students under active learning.

- **Students in active learning classes outperform those in traditional lectures on identical exams.** On average, students taught with active learning outperformed those taught by lectures by 6 percentage points on their exam. That’s the difference between bumping a B- to a B or a B to a B+.

In the authors’ view, the results of this study “raise questions about the continued use of traditional lecturing as a control in research studies, and support active learning as the preferred, empirically validated teaching practice in regular classrooms.”

**Concluding Observation**
Creating an effective active-learning environment takes a lot of work, and it requires more of both instructors and students. Yet, given mathematics’ importance as an “enabler” of STEM and other 21st century careers, as well as its support of advances across a broad array of fields, that effort is worth making.

Extensive research indicates that active learning confers substantial benefits for postsecondary mathematics students by engaging them in the learning process. It recognizes that students must do more than just listen. They must read, write, discuss, and grapple with problems in the search for solutions. And, they must be actively involved in higher-order thinking in the form of analysis, synthesis, and assessment.

**Endnotes**
3 Bhatia, A. (2014). “Active learning leads to higher grades and fewer failing students in science, math and engineering.” Science (April 12)