"Mathematics is an exciting and central human activity and, as a consequence, departments of mathematics and mathematical sciences have an obligation to position themselves at the core of their institutions. This includes active leadership for quantitative literacy.

Quantitative literacy does not need to be taught only by mathematicians any more than effective writing needs to be taught only by English professors. Mathematicians are not necessarily the best prepared to teach it. But each mathematics department has a responsibility to nurture and shape a meaningful program in quantitative literacy. More than a responsibility, the challenge to create such a program presents an opportunity for mathematics to take its rightful place of influence and importance at the heart of undergraduate education."

- David M. Bressoud, February 2004

As Chair of the Mathematical Association of America's Committee on the Undergraduate Program in Mathematics (CUPM) and presented in the foreword to *Achieving Quantitative Literacy*: *An Urgent Challenge for Higher Education* by Lynn A. Steen (2004)

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Teaching for Quantitative Reasoning

The creation of *TMM011 – Quantitative Reasoning* in the Ohio Transfer Module in December 2015 provided institutions of higher education with direction for developing a college-level quantitative reasoning course. Based on the recommendations in the CUPM Curriculum Guide (2004), the course should:

- Engage students in a meaningful intellectual experience
- Increase students' quantitative and logical reasoning abilities
- Improve students' ability to communicate quantitative ideas
- Encourage students to take other courses in the mathematical sciences
- Strengthen mathematical abilities that students will need in other disciplines

In order to achieve these goals, the learning outcomes for a quantitative reasoning course reflect a mathematical learning that is predominantly conceptual rather than procedural, and fosters a deeper understanding of the applicability and accuracy of standard mathematical skills and tools.

As such, developing student understanding of quantitative reasoning skills requires a classroom that promotes communication, collaboration, and persistence in problem solving. Hiebert and Grouws (2007) researched the development of conceptual understanding; among their findings, the most pertinent to college-level quantitative reasoning courses are:

1. Students need to be provided with ongoing opportunities to learn.

2. Deep understanding means forming connections between facts, ideas, and procedures in a social/cultural setting.

3. Making connections between mathematical concepts should be an explicit focus of students and teachers and is a product of active discourse.

4. Teachers provide opportunities to learn by allowing students to struggle with grasping important concepts.

5. Promoting conceptual understanding also means promoting skill fluency.

These elements are described in greater detail below.

Several authors have presented classroom strategies to promote conceptual learning that are based on stages or cycles defined by Shimizu (1996, 1999) as:

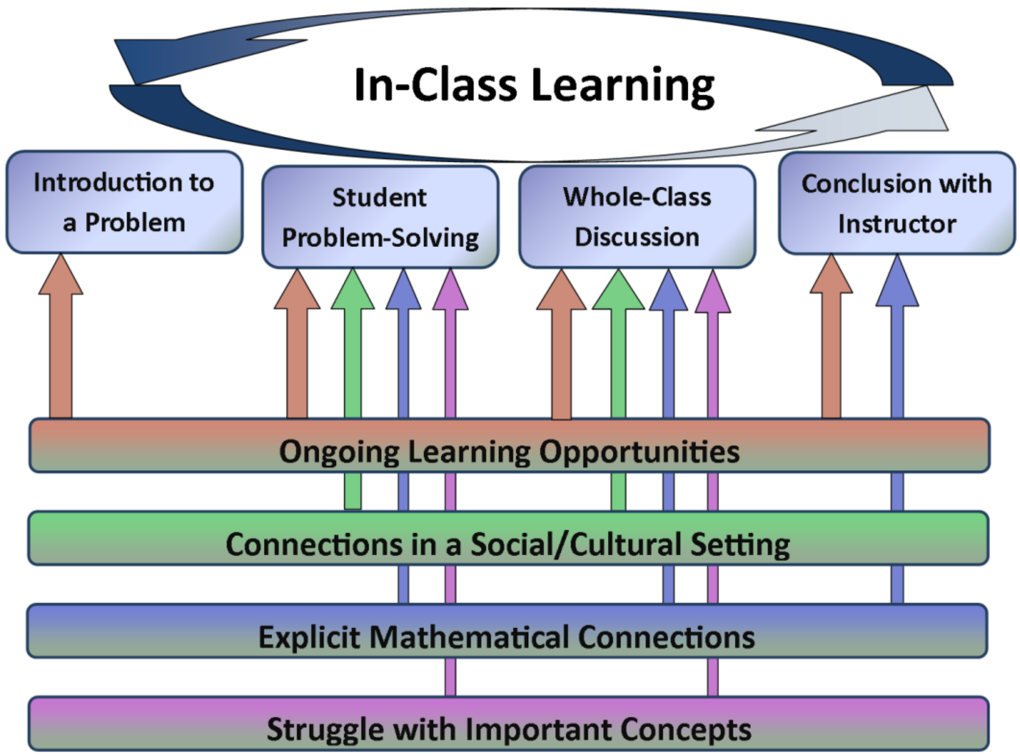
1. Introduction to a problem

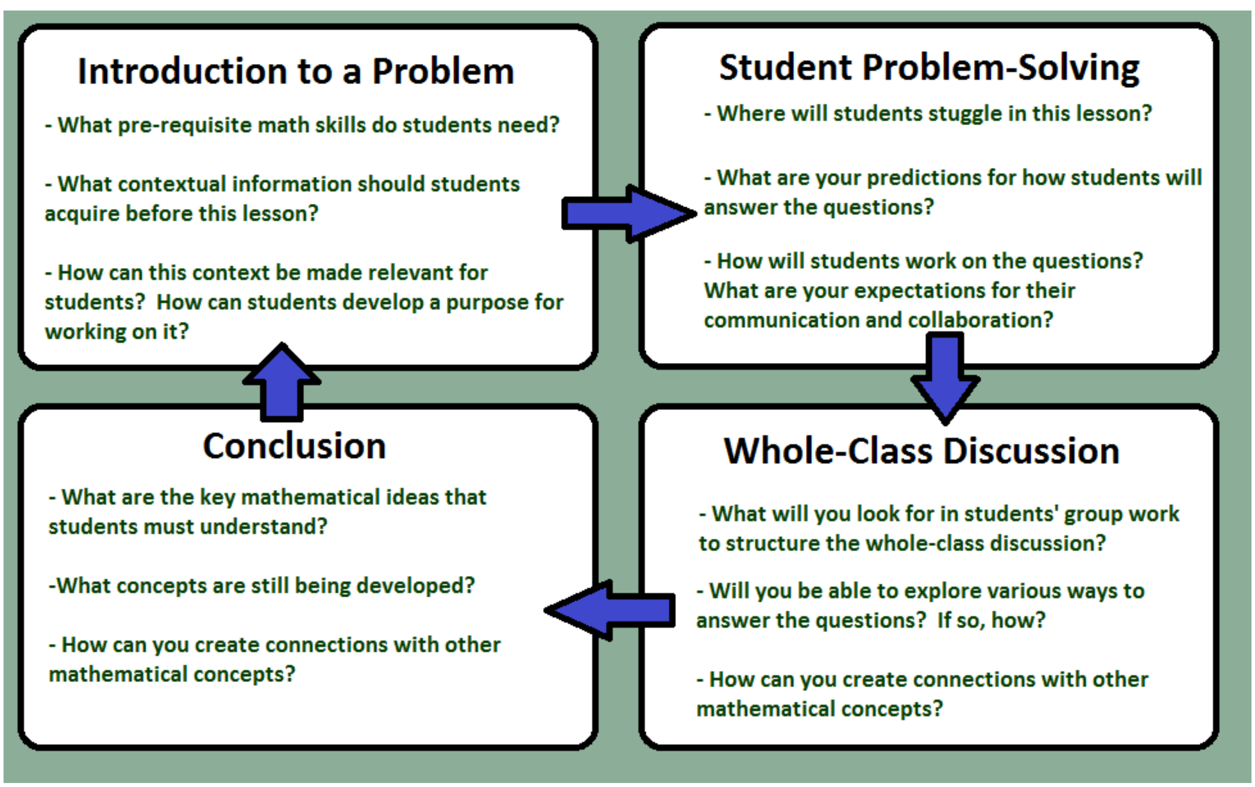
2. Problem solving by students

3. Whole-class discussion about ways to solve the problem

4. Conclusion facilitated by teacher

Givvin (2013) described these steps as part of a *problem cycle* that is applied once or multiple times within a class meeting. The stages within a problem cycle are designed to create a variety of learning opportunities for students. Launching the problem helps students to prepare to use prior knowledge and develops relevance and a reason to solve the problem. As students work through the problem in groups, the instructor supports students' struggle with the problem, focusing on thinking critically about the problem. A whole-class discussion provides opportunities to form explicit connections between mathematical concepts and an instructor-led conclusion supports those connections. Evans and Swan (2014) deeply explore the value of a whole-class discussion and sharing of student work for making explicit connections between mathematical ideas.


In-Class Learning
Introduction to a Problem
Student Problem-Solving
Whole-Class Discussion
Conclusion with Instructor
Ongoing Learning Opportunities
Connections in a Social/Cultural Setting
Explicit Mathematical Connections
Struggle with Important Concepts


Introduction to a Problem
- What pre-requisite math skills do students need?
- What contextual information should students acquire before this lesson?
- How can this context be made relevant for students? How can students develop a purpose for working on it?
Student Problem-Solving
- Where will students stuggle in this lesson?
- What are your predictions for how students will answer the questions?
- How will students work on the questions? What are your expectations for their communication and collaboration?
Conclusion
- What are the key mathematical ideas that students must understand?
-What concepts are still being developed?
- How can you create connections with other mathematical concepts?
Whole-Class Discussion
- What will you look for in students' group work to structure the whole-class discussion?
- Will you be able to explore various ways to answer the questions? If so, how?
- How can you create connections with other mathematical concepts?

Making Explicit Connections

"Students can acquire conceptual understanding of mathematics if…(instruction addresses) mathematical connections in an explicit and public way. Brophy (1999) described such teaching as infused with coherent, structured, and connected discussions of the key ideas of mathematics. This could include discussing the mathematical meaning underlying procedures, asking questions about how different solution strategies are similar to and different from each other, considering the ways in which mathematical problems build on each other or are special (or general) cases of each other, attending to the relationships among mathematical ideas, and reminding students about the main point of the lesson and how this point fits within the current sequence of lessons and ideas." (Hiebert & Grouws, 2007, p. 383)

Struggle with Important Concepts

"We use the word struggle to mean that students expend effort to make sense of mathematics, to figure something out that is not immediately apparent. We do not use struggle to mean needless frustration or extreme levels of challenge created by nonsensical or overly difficult problems. We do not mean the feelings of despair that some students can experience when little of the material makes sense. The struggle we have in mind comes from solving problems that are within reach and grappling with key mathematical ideas that are comprehendible but not yet well formed (Hiebert et al., 1996). By struggling with important mathematics we mean the opposite of simply being presented information to be memorized or being asked only to practice what has been demonstrated." (Hiebert & Grouws, 2007, p. 387)

Overcoming Student Misconceptions

"Because students possess deeply held beliefs and ideas regarding how mathematics is taught and learned, QL-friendly courses must overcome these conceptions by focusing on the applicability of the skills and the power of the mathematical processes that are to be developed during a semester-long course as well as practiced and refined in the student's life after the QL-course and post-college. Therefore, getting students to buy into the importance of gaining and strengthening the ability to reason quantitatively is a fundamental component they must gain from their experiences." (Dingman & Madison, 2010, p. 13).

"Although much of the mathematical and statistical content encountered in the course is generally taught in middle to early secondary grades, the embedding of the content in real-world contexts and the use of reasoning to determine solution strategies elevates the degree of sophistication for many students." (Dingman & Madison, 2010, p. 8)

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