WHAT DOES IT MEAN TO BE COLLEGE AND CAREER READY IN A COMMON CORE WORLD?

MATHEMATICS ACROSS THE P-16 CURRICULUM

Fred Dillon – Fred.Dillon@Ideastream.org

Thinking about Math

nctm.com

The following problem is from *Teaching Mathematics: A Sourcebook of Aids, Activities, and Strategies* by Sobel and Maletsky,

1999, 3rd ed. Boston, MA: Allyn and Bacon.

Thinking about Math

A customer enters a store and purchases a pair of slippers for $5, paying for the purchase with a $20 bill.

The merchant, unable to make change, asks the grocer next door to change the bill. The merchant then gives the customer the slippers and $15 change.

A Counterfeit bill

After the customer leaves, the grocer discovers that the $20 bill is counterfeit and demands that the shoe-store owner make good for it.

The shoe-store owner does so, and by law is obligated to turn the counterfeit bill over to the FBI. How much does the shoe-store owner lose in this transaction?

What Skills Did You Use?

What should students take from this problem?

What should educators take from this problem?

CCSSM

- Common Core State Standards for Mathematics, Adopted by Ohio in June 2010
- The State Board of Education adopted Ohio's New Learning Standards in Mathematics in June 2011.

http://education.ohio.gov/Topics/Ohio-s-New-Learning-Standards/Mathematics

Standards for Mathematical Practice

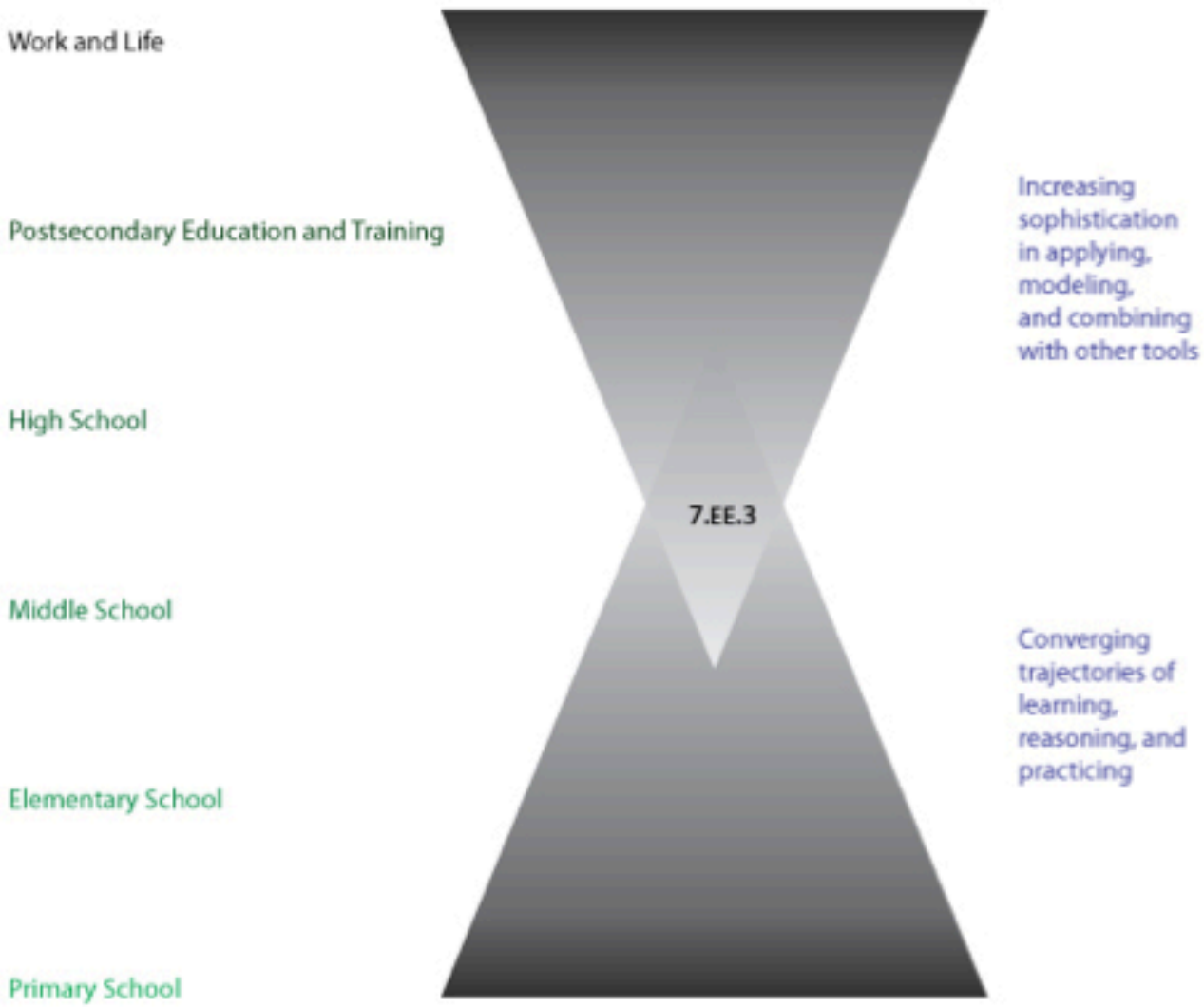
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Common Core State Standards for Mathematics, 2010, pg. 6-12

CCSSM – College and Career Ready

Some of the highest priority content for college and career readiness comes from Grades 6-8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume

Common Core State Standards, 2010, pg. 84



From the PARCC Model Content Frameworks Appendix A, http://www.parcconline.org/parcc-content-frameworks.

College and Career Ready

College and career readiness means that a high school graduate has the knowledge and skills in English and mathematics needed to qualify for and succeed in postsecondary job training and/or education necessary *without remediation.*

http://www.achieve.org/college-and-career-readiness

Ohio Recommended Mathematical Expectations for College Readiness

This document "represents the minimal expectations." Extensions of the material are encouraged.

College Readiness and Remediation Free Standards Recommendations: Report to the Ohio College Readiness Advisory Committee, 2012

Ohio Recommended Mathematical Expectations for College Readiness

Alignment with CCSSM

Expectation 1: Mathematical Processes

Essentially, these are the Student Mathematical Processes (SMP), extending those to include connections among topics and a stronger statement on mathematical communication

College Readiness and Remediation Free Standards Recommendations: Report to the Ohio College Readiness Advisory Committee, 2012

Ohio Recommended Mathematical Expectations for College Readiness

Alignment with CCSSM

Expectation 2: Number and Operations

Adds computation and estimation with Real numbers

College Readiness and Remediation Free Standards Recommendations: Report to the Ohio College Readiness Advisory Committee, 2012, pg. 30

Ohio Recommended Mathematical Expectations for College Readiness

Alignment with CCSSM

Expectation 3: Algebra

Combines many outcomes from Algebra and Function

College Readiness and Remediation Free Standards Recommendations: Report to the Ohio College Readiness Advisory Committee, 2012, pg. 30-32

Ohio Recommended Mathematical Expectations for College Readiness

Alignment with CCSSM

Expectation 4: Geometry

Fewer guidelines on transformations and their use in proof. Includes measurement standards that align with SMP 6 (Precision) and some of the Numbers and Operation Standards.

College Readiness and Remediation Free Standards Recommendations: Report to the Ohio College Readiness Advisory Committee, 2012, pg. 32-33

Ohio Recommended Mathematical Expectations for College Readiness

Alignment with CCSSM

Expectation 5: Probability and Statistics

Combines univariate and bivariate data in the outcomes, not as specific as Common Core with analysis of studies and inference.

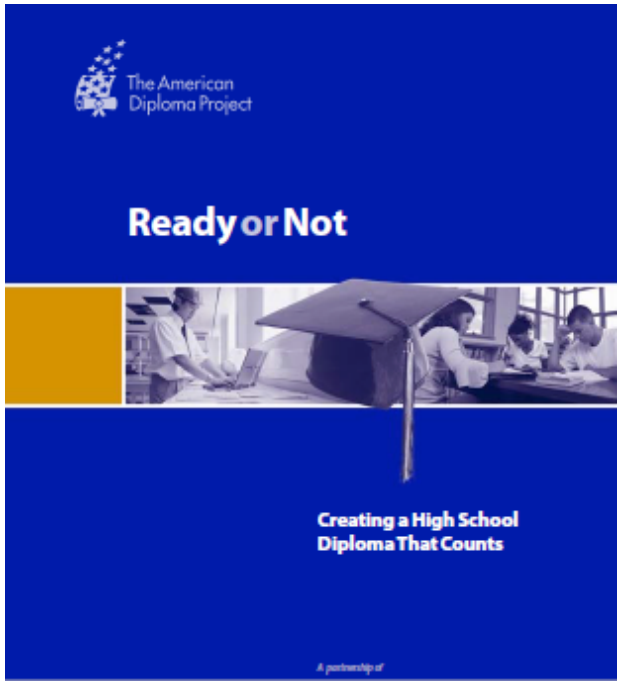
College Readiness and Remediation Free Standards Recommendations: Report to the Ohio College Readiness Advisory Committee, 2012, pg. 34

College and Career Ready

"Simply put, 'college and career readiness' is the umbrella under which many education and workforce policies, programs and initiatives thrive. From high-quality early education and strong, foundational standards in elementary school to rigorous career and technical education programs and college completion goals, college and career readiness is the unifying agenda across the P-20 education pipeline."

http://www.achieve.org/college-and-career-readiness

American Diploma Project



- Started in 2004 to identify a common core of English and mathematics academic knowledge and skills.
- *Education Week* later named *Ready or Not* one of the most 12 influential research studies.

http://www.achieve.org/history-achieve

American Diploma Project

A P-20 CONTINUUM OF ACADEMIC READINESS

I. School Readiness	II. High School Readiness	III. College and Career Readiness (CCR)	IV. Postsecondary Success
a. Kindergarten Readiness	a. Mathematics in Grade 5	a. Cohort Graduation Rate	a. Remediation
b. Reading in Grades K-2	b. Course Failure in Mathematics or English/language arts in grade 6	b. CCR Diploma	b. Early Postsecondary Success
c. Reading in Grade 3	c. Completion of 8th Grade Mathematics or Algebra I with a "C" or Higher by the End of the 8th grade	c. CCR Assessments	c. Postsecondary Completion
d. Mathematics in Grade 3		d. Earning College Credit in High School	
		e. Career Readiness	

Achieve, *American Diploma Project* – Student Readiness Indicators

Pre-K

- Know number names and count sequence (count 1-10)
- Sort quantities and objects (greater than, categories)
- Understand addition as “putting together” and subtraction as “taking apart” or “taking from”
- Identify and describe two dimensional shapes (circle, triangle, rectangle, square)

These suggestions from Maryland SDE

There are no Pre-K CCSSM standards because funding for Pre-K is not the same across all states

Kindergarten

Suggested areas of focus

- Representing, relating, and operating on whole numbers with sets of numbers
- Describing shapes and space
- Cardinality

More learning time in K should be devoted to number than other topics.

All Focus areas adapted from *Common Core Mathematics in a PLC at Work series*, Solution Tree, 2013

First Grade

Four critical areas:

- Developing understanding of addition and subtraction, and strategies for addition and subtraction within 20
- Whole number relationships and place value
- Linear measurement and measuring lengths
- Composing and decomposing geometric shapes

All Focus areas adapted from *Common Core Mathematics in a PLC at Work series*, Solution Tree, 2013

Second Grade

Four critical areas:

- Extending understanding of base-ten notations
- Building fluency with addition and subtraction
- Using standard units of measure
- Describing and analyzing shapes

All Focus areas adapted from *Common Core Mathematics in a PLC at Work series*, Solution Tree, 2013

Third Grade

Four critical areas

- Strategies for multiplication and division within 100
- Develop understanding of fractions, especially unit fractions
- The structure of rectangular arrays and of area
- Describe and analyze 2D shapes

All Focus areas adapted from *Common Core Mathematics in a PLC at Work series*, Solution Tree, 2013

Fourth Grade

Three critical areas

- Developing understanding of multi-digit multiplication and of dividing multi-digit dividends.
- Develop understanding of fractions equivalence, adding and subtraction with like denominators, and multiplying by a whole number
- Geometric figures can be analyzed by properties such as symmetry, parallel sides, angle measures, and/or perpendicular side.

Fifth Grade

Three critical areas

- Developing fluency with addition and subtraction of fractions, understanding multiplication of fractions, division of fractions in limited cases (involving whole numbers and unit fractions)
- Extending division to two digit divisors, integrating decimals into place value and developing understanding and fluency of decimal operations
- Developing understanding of volume

Sixth Grade

Four critical areas

- Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems
- Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers
- Writing, interpreting and using expressions, and equations
- Developing understanding of statistical thinking

Seventh Grade

Four critical areas

- Developing understanding of operations with rational numbers and working with expressions and linear equations
- Developing understanding of and applying proportional relationships
- Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume
- Drawing inferences about populations based on samples.

Eighth Grade

Three critical areas

- Formulating and reasoning about expressions and equations, modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations.
- Grasping the concept of a function and using functions to describe quantitative relationships
- Analyzing two- and three - dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem (irrational numbers)

Major Focus Areas in Common Core

Grades K-2 Number and Base 10

Cardinality
Whole number operations, creating and understanding operations
Number used in contexts

Adapted from *Common Core Mathematics in a PLC at Work series*, Solution Tree, 2013

Major Focus Areas in Common Core

Grades 3-5 Fraction and Number

Begins with multiple representations of fractions and whole numbers.

Computation is taught for deep understanding and not as a complex algorithm

Fractions are taught in context of applications and problem solving

Adapted from *Common Core Mathematics in a PLC at Work series*, Solution Tree, 2013

Mr. Edmunds shared 12 pencils among his four sons as follows:

- Alan received $\frac{1}{3}$ of the pencils.
- Bill received $\frac{1}{4}$ of the pencils.
- Carl received more than 1 pencil.
- David received more pencils than Carl.

Part A

On the number line, represent the fraction of the total number of pencils that was given to both Alan and Bill combined. Use the buttons on the right to increase or decrease the number of equal sections on the number line.

More Tick Marks

Fewer Tick Marks

Part B

What fraction of the total number of pencils did Carl and David **each** receive? Justify your answer.

http://www.parcconline.org/samples/item-task-prototypes

Major Focus Areas in Common Core

Grades 6-7 Ratio and Proportion

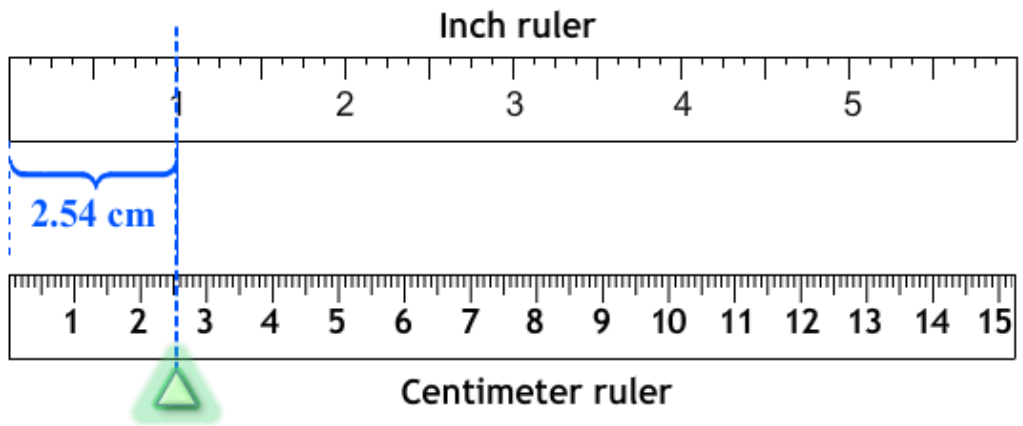
Connects to Fraction – equivalent sets of fractions, ratio boxes

Multiple approaches – strip diagrams, parallel number lines, sets of equivalent ratios, graphing

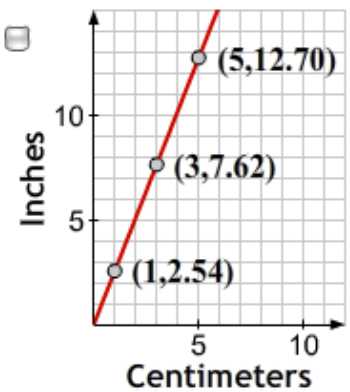
Based on application and problem solving

Concept more important that learning an algorithm

Adapted from *Common Core Mathematics in a PLC at Work series*, Solution Tree, 2013



If we say that the relationship between the number of inches and the number of centimeters is exact, which of the following correctly represents the relationship? Select all that apply.

- ☐ $i = 2.54c$, where i stands for the number of inches and c stands for the number of centimeters
- ☐ $c = 2.54i$, where c stands for the number of centimeters and i stands for the number of inches
- ☐ The ratio of centimeters to inches is 1 to 2.54.
- ☐ The ratio of centimeters to inches is 2.54 to 1.
- ☐ 

http://www.ccsstoolbox.com/parcc/PARCCPrototype_main.html

Major Focus Areas in Common Core

Grades 8 and up – Function

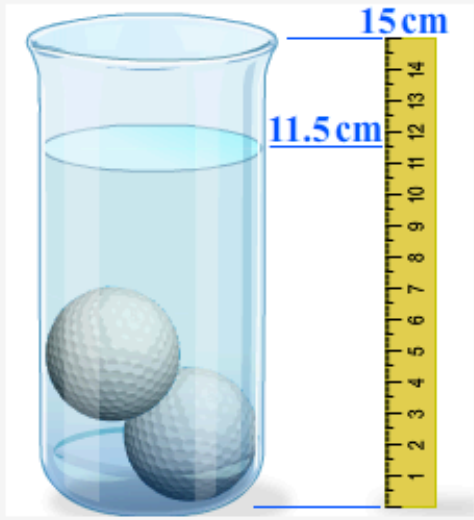
Grows directly from the Proportion Focus

Requires multiple representations – technology, graphs, symbols, tables, verbal, physical

Based on applications

Adapted from *Common Core Mathematics in a PLC at Work series*, Solution Tree, 2013

Tom is doing an experiment adding golf balls to a glass jar containing water. The picture and the table show what happens to the height of the water as Tom adds golf balls.



Number of golf balls, x	Height of water in centimeters, y
0	9.0
1	10.2
2	11.5
3	12.7
4	13.8

There are several ways that Tom could modify the conditions of his experiment.

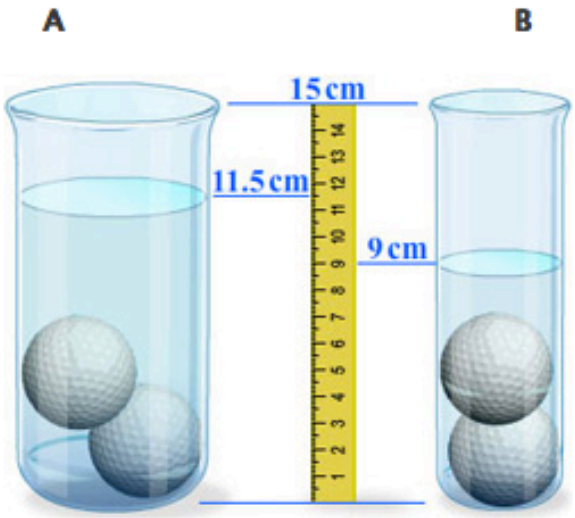
What modifications would increase the rate of change in the height of the water level with respect to the number of golf balls? Select all that apply.

- [ ] Use larger golf balls
- [ ] Decrease the diameter of the glass jar
- [ ] Drop the golf balls into the glass jar at a faster rate
- [ ] Add 5 cm of water to the glass jar
- [ ] Drop the golf balls into the glass jar two at a time

http://www.ccsstoolbox.com/parcc/PARCCPrototype_main.html

Data from Experiment with Glass Jar A

Number of golf balls, x	Height of water in centimeters, y
0	9.0
1	10.2
2	11.5
3	12.7
4	13.8



Tom forgot to write down the initial height of the water in glass jar B, but he measured the water height at 9 centimeters after adding two golf balls.

Question a: When Tom creates graphs of the data from both experiments, how will the y-intercepts of the graphs be different for glass jar A versus glass jar B? Explain how you know.

Question b: How will the rate of change in the experiment using glass jar B be different than the rate of change in the experiment using glass jar A? Explain how you know.

Question c: Suppose glass jar B has a water height of 5 centimeters with no golf balls, and the water height increases at a rate of 2 centimeters per golf ball added. Tom continues to add golf balls to each glass jar. He discovers that there is a number of golf balls at which the height of the water in each glass jar is the same. How many golf balls will be in each jar when the water in each reaches the same height?

http://www.ccsstoolbox.com/parcc/PARCCPrototype_main.html

More Information

ACT has detailed score ranges for five of the CCSSM content areas (there are none for Modeling).

Topics in the flow to...	Score Range 13–15	Score Range 16–19	Score Range 20–23	Score Range 24–27	Score Range 28–32	Score Range 33–36
Functions (F)	**AF 201.** Solve problems in one or two steps using whole numbers and using decimals in the context of money **F 201.** Extend a given pattern by a few terms for patterns that have a	**AF 301.** Solve routine one-step arithmetic problems using positive rational numbers, such as single-step percent **AF 302.** Solve some routine two-step arithmetic problems	**AF 401.** Solve routine two-step or three-step arithmetic problems involving concepts such as rate and proportion, tax added, percentage off, and estimating by using a given average value in place of actual	**AF 501.** Solve multistep arithmetic problems that involve planning or converting common derived units of measure (e.g., feet per second to miles per hour) **AF 502.** Build functions and write	**AF 601.** Solve word problems containing several rates, proportions, or percentages **AF 602.** Build functions and write expressions, equations, and inequalities for common algebra	**AF 701.** Solve complex arithmetic problems involving percent of increase or decrease or requiring integration of several concepts (e.g., using several ratios, comparing percentages, or comparing averages)

http://www.act.org/standard/planact/math/

More Information

There are details on changes since 2005.

ACT CCRS Improvements 2014: Algebra and Functions (AF) or Functions (F), Score Range 13–15

2014 ACT College and Career Readiness Standard	Corresponding 2005 College Readiness Standard
AF 201. Solve problems in one or two steps using whole numbers and using decimals in the context of money **F 201.** Extend a given pattern by a few terms for patterns that have a constant increase or decrease between terms	**BOA 202.** Solve problems in one or two steps using whole numbers

http://www.act.org/standard/planact/math/

More Information

The College Board also has information on College and Career Readiness.

"The SAT Benchmark score of 1550 is associated with a 65% probability of obtaining a first-year GPA of B- or higher at a four-year college.

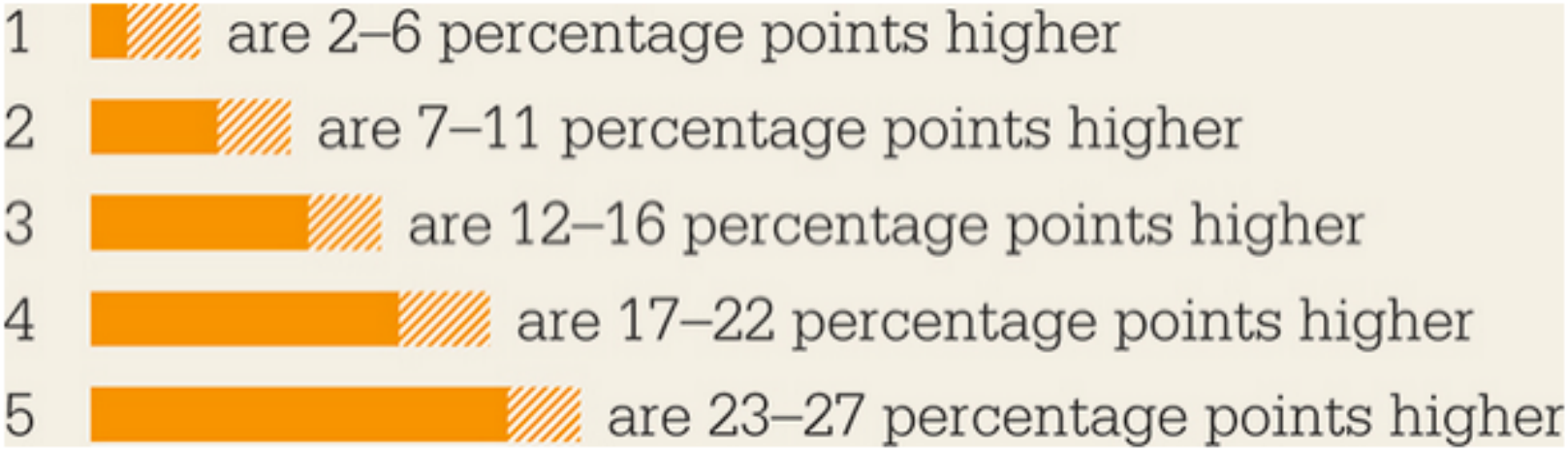
Only **42.6%** of SAT takers in the class of 2014 met the SAT College and Career Readiness Benchmark. This number has remained virtually unchanged over time."

https://www.collegeboard.org/program-results/2014/sat

More Information

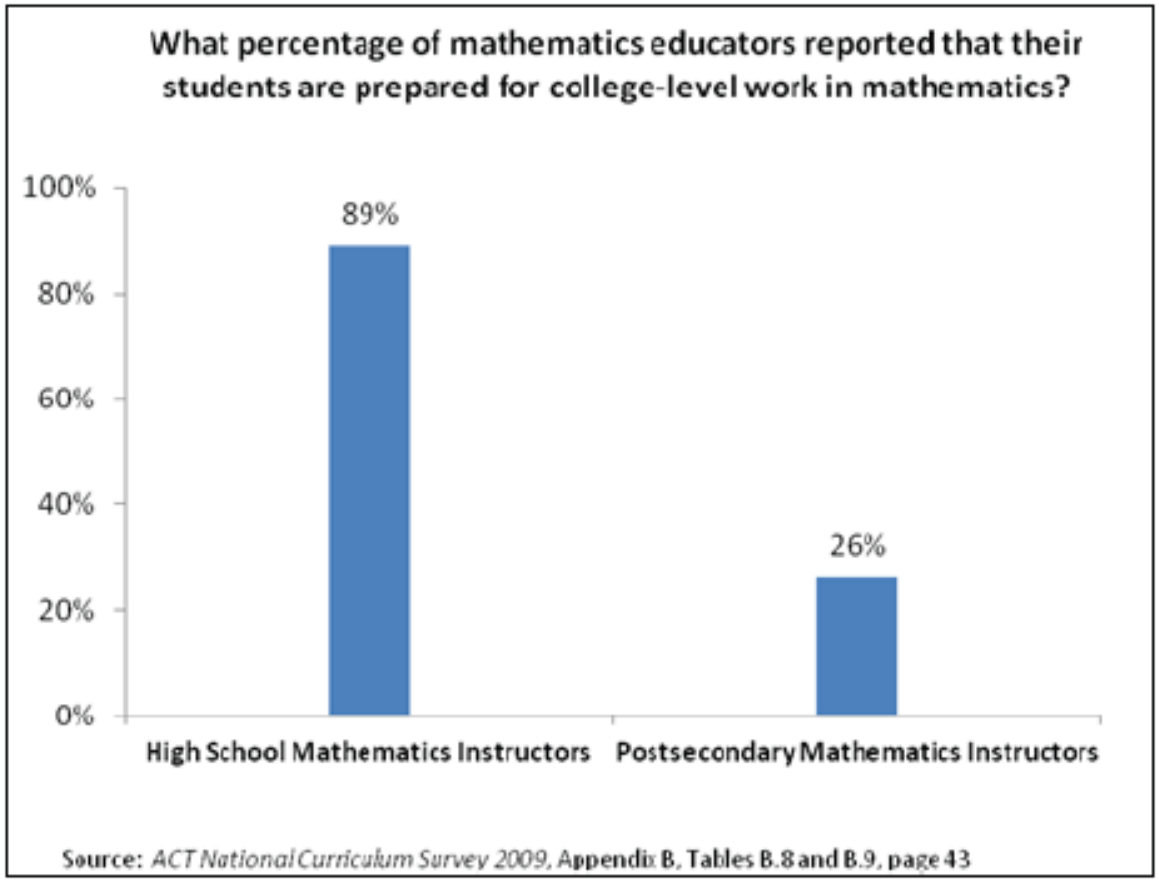
Ohio is below the national average for student AP participation (16% Ohio, 21.9% for the U.S.) and performance (13.2% score a "passing score" nationally, with Ohio at 10.3%)

Probability of on-time college graduation compared to academically matched peers without AP experience.



https://www.collegeboard.org/program-results/2014/sat

College Ready



ACT National Curriculum Survey 2009, Appendix B, page 43

College & Career Ready

Postsecondary instructors in [PARCC] surveys tend to value mastery of fundamentals over broad topic coverage.

…“ being able to approach problems in the ways described in the Standards for Mathematical Practice.”

…“being able to solve complex problems using securely held knowledge and skills. The ability to flexibly apply what one already knows to a nonroutine or complex problem is an important aspect of readiness for college and careers.

PARCC MODEL CONTENT FRAMEWORKS
MATHEMATICS
GRADES 3–11, 2011 page 76

CCSSM 9-12 Conceptual Categories

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

CCSSM material drawn from *Common Core State Standards for Mathematics*, CCSSO and NGA 2010, 2011

CCSSM 9-12 Strands

The content traditionally associated with Algebra I and Algebra II is split into two content strands – Algebra (symbols, expressions, equations) and Function (graphing, rates of change)

9-12 Strands

Geometry has increased emphasis on transformations - and looks at transformations as functions at the HS level

Statistics and Probability have increased emphasis.

Number and Operation

Extend the properties of exponents to rational number exponents

Use properties of rational and irrational numbers

Algebra

Write expressions in equivalent forms to solve problems (Arithmetic with polynomials)

Interpret the structure of expressions

Understand the relationship of factors with zeros of polynomials

Algebra

Solve equations, inequalities and systems – both symbolically and graphically

Functions

Connect to expressions, equations, modeling and coordinates

Use different representations

Apply in contexts

Functions

Linear, quadratic, exponential models

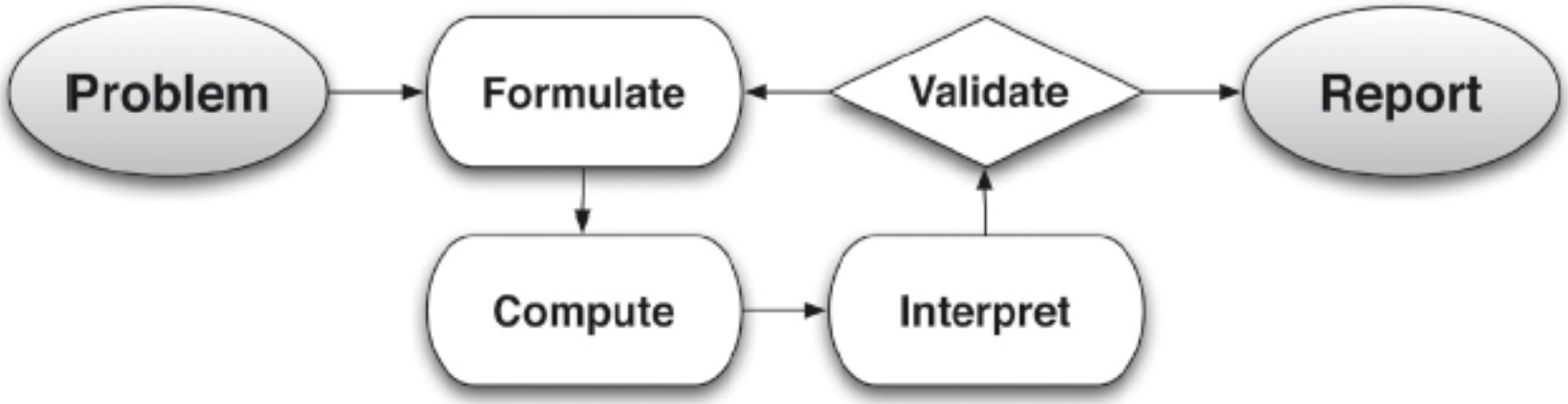
Piecewise, step functions, square root

Recursive and Closed form

Modeling

Being able to apply mathematics to modeling is a separate strand at the HS level.

Modeling



Common Core State Standards for Mathematics, 2010

Geometry

Transformations, dynamic geometry, apply through modeling

Statistics and Probability

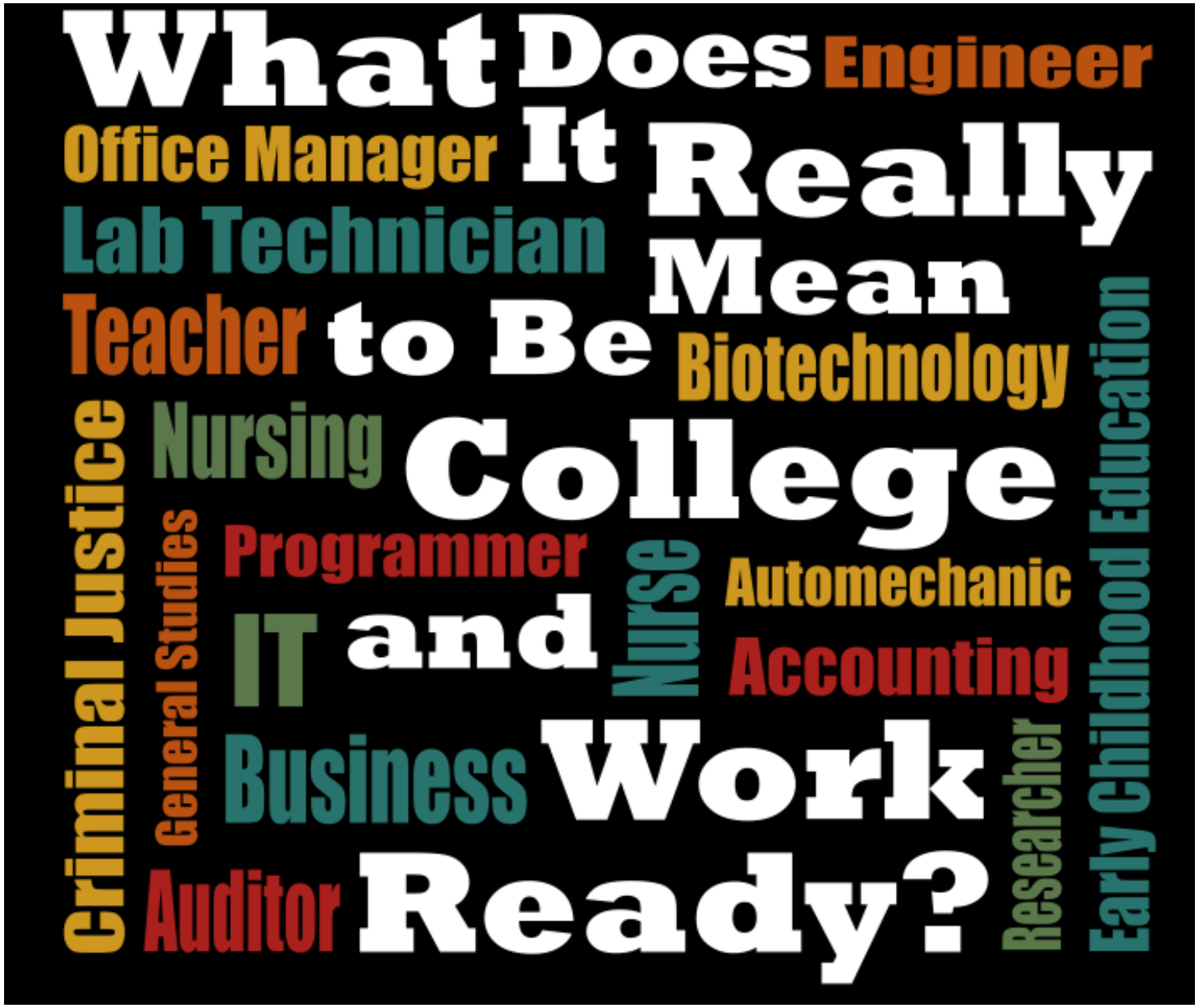
- Summarize, represent, and interpret data from one or two variables
- Interpret linear models

Statistics and Probability

Make inferences and justify conclusions from sample surveys, experiments and observational studies

Statistics and Probability

Understand independence, conditional probability, compound events



National Center on Education and the Economy, 2013

Transition from High School

"[Standards must], over time, be raised —greatly raised in fact—but it is very important to bear in mind that a large fraction of high school graduates cannot now do the work required of them in the first year of the typical community college program."

Executive Summary, National Center on Education and the Economy, 2013, page 8

Key Finding

"A very high priority should be given to the improvement of the teaching of proportional relationships…"

Executive Summary, National Center on Education and the Economy, 2013, page 12

Key Finding

More emphasis is needed for "Percent, graphical representations, functions, and expressions and equations, including their application to concrete practical problems"

Executive Summary, National Center on Education and the Economy, 2013, page 12

Key Finding

"Mastery of Algebra II is widely thought to be a prerequisite for success in college and careers.

Our research shows that that is not so."

Executive Summary, National Center on Education and the Economy, 2013, page 12

Key Finding

"The most demanding mathematics courses typically required of community college students are those required by the mathematics department, not the career major"

Executive Summary, National Center on Education and the Economy, 2013, page 12

Remediation?

"The content of the first year mathematics courses … is typically the content usually associated with Algebra I, some Algebra II…"

Executive Summary, National Center on Education and the Economy, 2013, page 12

What Used to Be

There is an "assumption that all students should be expected to be proficient in the sequence of courses leading to calculus"

Executive Summary, National Center on Education and the Economy, 2013, page 13

What Does It Really Mean to Be College and Career Ready?

Learning should be about "analytical skills, writing ability, ability to put together solutions to problems the student has not seen before"

Executive Summary, National Center on Education and the Economy, 2013, page 14

Resource

American Mathematical Association of Two Year Colleges (AMATYC) – *Crossroads*

Suggestions and standards for introductory and developmental mathematics courses at two year institutions

Reports from the MAA/NCTM

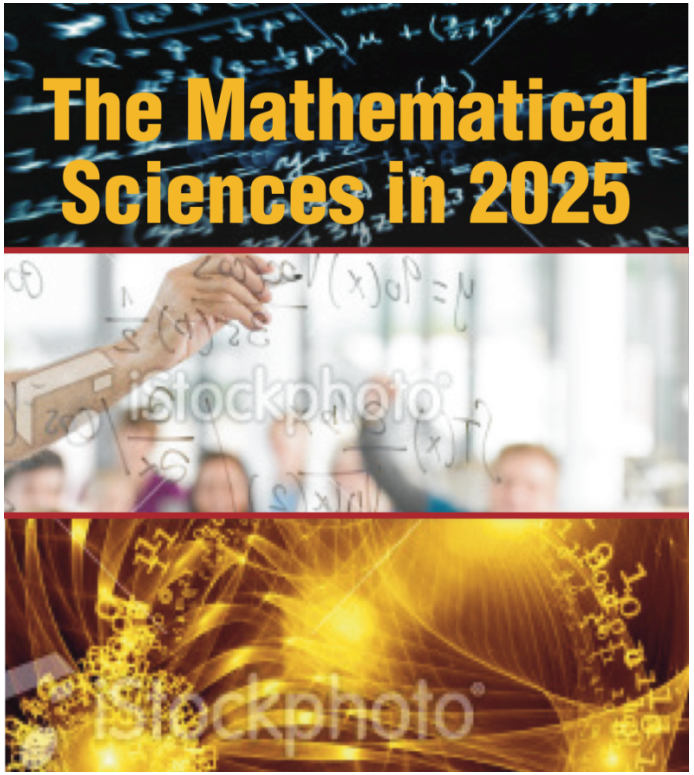
- "Meeting the Challenges of Postsecondary Education in the Mathematical Sciences," (Bressoud, Friedlander and Levermore), *MAA Focus*, February-March 2014
- Endless Algebra is the Deadly Pathway from High School Mathematics to College Mathematics (Shaughnessy, NCTM President's Message, Feb., 2011)

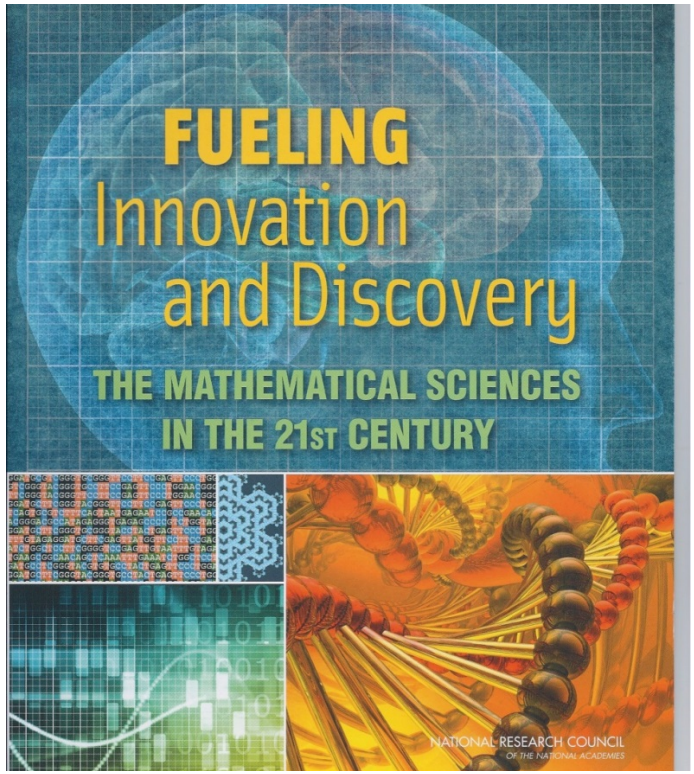
What are some alternative transitions?

Rethinking Postsecondary Mathematics, Ohio Mathematics Initiative, March 2014

"Develop high-quality entry-level courses and pathways connected to coherent programs of study for students majoring in (1) mathematics, (2) other mathematics-intensive majors, and (3) majors that are not mathematics intensive."

College and Career Readiness





College and Career Readiness

- Common Vision for Undergraduate Mathematics in 2025 (MAA-AMATYC-AMS-ASA-SIAM)
- Transforming Post-Secondary Education in Mathematics, TPSE Math (AMS-MAA-SIAM)
- Modeling across the Curriculum, MaC (SIAM-ASA-MAA)
- *Guidelines for Assessment and Instruction in Mathematical Modeling Education* (GAIMME)—SIAM, April 2016
- Preparation for Industrial Careers in Mathematics, PIC-Math (MAA-SIAM)
- Investing in the Next Generation through Innovative and Outstanding Strategies, INGenIOuS (MAA-ASA-AMS-SIAM)
- Guidelines for Assessment and Instruction in Statistics Education GAISE (ASA but also being used collaboratively with SIAM in developing an analogous Mathematical Modeling, GAIMME report)

Diane J. Briars, April 2015

Potential Changes in High School Mathematics

- Increased emphasis on statistics and modeling
- Increased emphasis on conceptual understanding, including ratios and proportional relationships.
- Slowing down the rush to calculus

Thank You

Thank you for your participation!

Fred.Dillon@Ideasteam.org