TMM003 - TRIGONOMETRY (Revised March 21, 2017)

Typical Range: 3-4 Semester Hours

Recommendation: This course should significantly reflect the spirit of the Mathematical Association of America's Committee on the Undergraduate Program in Mathematics (CUPM), Curriculum Renewal Across the First Two Years (CRAFTY), College Algebra Guidelines.

College Algebra provides students a college-level academic experience that emphasizes the use of algebra and functions in problem solving and modeling, where solutions to problems in real-world situations are formulated, validated, and analyzed using mental, paper-and-pencil, algebraic and technology-based techniques as appropriate using a variety of mathematical notation. Students should develop a framework of problem-solving techniques (e.g., read the problem at least twice; define variables; sketch and label a diagram; list what is given; restate the question asked; identify variables and parameters; use analytical, numerical and graphical solution methods as appropriate; and determine the plausibility of and interpret solutions).
– Adapted from the MAA/CUPM CRAFTY 2007, College Algebra Guidelines

To qualify for TMM003 (Trigonometry), a course must achieve all of the following essential learning outcomes listed in this document (marked with an asterisk). These make up the bulk of a Trigonometry course. Courses that contain only the essential learning outcomes are acceptable from the TMM003 review and approval standpoint. It is up to individual institutions to determine further adaptation of additional course learning outcomes of their choice to support their students' needs. The Sample Tasks are suggestions/ideas for types of activities that could be used in the course. The Sample Tasks are not requirements.

1. **Functions:** Successful Trigonometry students demonstrate a deep understanding of periodic functions. This includes trigonometric functions whether they are described verbally, numerically, graphically, pictorially, geometrically, or algebraically.

   The successful Trigonometry student can:

   1a. Analyze functions. Routine analysis includes discussion of domain, range, zeros, and general function behavior (increasing, decreasing, extrema, etc.), as well as periodic characteristics such as period, frequency, phase shift, and amplitude with emphasis on functions derived from the geometry of the unit circle. In addition to performing rote processes, the student can articulate reasons for choosing a particular process, recognize function families and anticipate behavior, and explain the implementation of a process (e.g., why certain real numbers are excluded from the domain or range of a given function).*

   Sample Tasks:

   - The student determines the domain and range of a function described algebraically and gives reasons for restrictions.
   - The student determines the domain and range of a function given its graph.

- The student can explain to a peer how to evaluate a given piecewise-defined function.
- The student recognizes and accurately represents asymptotic behavior on the graph of a trigonometric function.
- The student can explain the difference between the quantities $f(x+h)$ and $f(x) + h$.
- The student can explain the difference between $f(x)+f(x)$ and $f(x) + f(y)$.
- The student can explain the inverse relationships for trigonometric functions, as well as explain domain and range restrictions and interpret geometrically.
- Analyze compositions involving known functions:
$$\log(\sin(t)), e^{\cos(x)}, \sqrt{\cos(\theta)}, \sin\left(\frac{1}{\varphi}\right), e^{-t}\sin(2u).$$
- The student can explain why $\frac{\pi}{2}$ is excluded from the domain of the tangent function.
- The student can explain the difference between $sin(2x)$ and $2\ sin(x)$.

1b. Convert between different representations of a function.*

Sample Tasks:

- The student translates a function from a verbal description to an algebraic description to determine its domain.
- The student constructs a table or a graph to approximate the range of a function described algebraically.
- The student graphs a piecewise-defined function to determine intervals over which the function is increasing, decreasing, or constant.
- The student can formulate a possible equation for a function given a graph.
- The student can verbalize the function represented variously as: $y = \sin(x)$, $h(t) = \sin(t)$, and $\{(u, \sin(u)) : u\ is\ a\ real\ number\}$ as 'the sine function.'
- The student uses knowledge of end or asymptotic behavior to adjust the viewing window of a graphing utility.
- The student suspects symmetry about an axis, the origin, or another line by analyzing a graph and proves the symmetry analytically.
- The student can discuss domain and range of functions defined implicitly: $\sin(x)\cos(y) = \frac{1}{2}$.

1c. Perform operations with functions including addition, subtraction, multiplication, division, composition, and inversion; connect properties of constituent functions to properties of the resultant function; and resolve a function into a sum, difference, product, quotient, and/or composite of functions.*

Sample Tasks:

- Through extending the graphs of quotient functions from rational functions to include trigonometric functions, the student constructs the graph of $y = \frac{\sin(3x)}{\cos(2x)}$.
- The student can verbally describe the relationship between the graph of $y = f(t)$ and each of $y = f(|t|)$ and $y = |f(t)|$.

- Given an algebraic description for f and a graph for g, the student can determine values for the sum, difference, product, quotient, and composition of f and g.
- Given the graph of f and g, the student can determine the domain of $\frac{f}{g}$ and $f \circ g$.
- Given formulas for $f(\theta)$ and $(f \circ g)(\theta)$, the student can create a formula for $g(\theta)$.
- The student can explain how to determine a formula for the composition of two piecewise-defined functions.
- Given the graph/formula of a function, the student can determine if the function is invertible and, if so, graph the inverse or create formula.
- The student can find functions f, g, and h so that
$$F(t) = \sqrt{\frac{3\cos(t) - 4}{\cos(t) + 1}} = \left(f \circ \left(\frac{g}{h}\right)\right)(t).$$
- Given the graph of a function f, the student can graph $y = 3f(1 - x) + 2$.
- Given the graphs of two functions, the student can determine if they appear to be related by a sequence of linear transformations.
- The student can interpret $e^{-y}\ sin(3y)$ as a sinusoid with an exponentially decaying amplitude (envelope).

2. **Geometry:** Successful Trigonometry students demonstrate a deep understanding of the measurements of right triangles, right triangles as building blocks of general triangles, and right triangles as a bridge between circular measurements and rectangular measurements.

The successful Trigonometry student can:

2a. Analyze angles. Routine analysis of angle measurements, units, and arithmetic.*

Sample Tasks:

- The student can measure drawings of angles using degrees and radians and convert between the two systems.
- The student can estimate measurements of angles and sketch angles with given measurements.
- The student can extend absolute measurements to the plane adding a positive and negative direction.

2b. Analyze right triangles. Routine analysis of side lengths and angle measurements using trigonometric ratios/functions, as well as the Pythagorean Theorem.*

Sample Tasks:

- The student can solve right triangles numerically using trigonometric ratios and relationships.
- The student can compare similar triangles numerically.
- The student can describe relationships within or between right/similar triangles algebraically using trigonometric ratios and relationships.

2c. Analyze general triangles. Routine analysis of side lengths and angle measurements using trigonometric ratios/functions, as well as other relationships.*

Sample Tasks:

- The student can solve general triangles using trigonometric ratios and relationships including laws of sine and cosine.
- The student can compare similar triangles.
- The student can compute length and angle measurements inside complex drawings involving multiple geometric objects.
- The student can algebraically describe relationships inside complex drawings involving multiple geometric objects.

3. Equations and Inequalities: Successful Trigonometry students are proficient at solving a wide array of equations and inequalities involving trigonometric functions.

The successful Trigonometry student can:

3a. Recognize function construction/algebra as it appears in equations and inequalities and choose an appropriate solution methodology for a particular equation or inequality, as well as communicate reasons for that choice.*

Sample Tasks:

- The student can summarize a solution strategy for a given problem verbally, without actually solving the problem.
- The student can solve an equation by factoring and explain the connection to the zero product property of real (complex) numbers.
- The student can explain the steps taken to construct a sign diagram and use a sign diagram to solve an inequality.
- The student can solve an equation involving piecewise-defined functions.
- The student can solve $2\,sin^2(t) + 7\sin(t) - 4 = 0$ on a given interval.
- The student can solve $log_4(\sin(t)) + log_4(2\sin(t) + 7) = 1$ on a given interval.

3b. Use correct, consistent, and coherent notation throughout the solution process to a given equation or inequality.*

Sample Tasks:

- The student is comfortable with given function and variable names.
- The student can choose meaningful function and variable names given a situation to model.

3c. Distinguish between exact and approximate solutions and which solution methodologies result in which kind of solutions.*

Sample Tasks:

- The student lists the exact values of the irrational zeros of a quadratic function and uses decimal approximations to sketch the graph.
- The student recognizes the need to approximate the solutions to $\sin(x)\cos(y) = \frac{1}{2}$ and uses a graphing utility to do so.

3d. Demonstrate an understanding of the algebraic, functional, and geometric views of equation solutions. Solutions to equations can simultaneously serve multiple purposes by representing numbers satisfying an equation, zeros of a function, and intersection points of two curves.*

Sample Tasks:

- The student solves an equation algebraically and verifies the solution using a graphing utility.
- Given the graphs of two functions f and g, the student can approximate solutions to $f(x) = g(x)$.

3e. Cite domain restrictions resulting from solution decisions and situation restrictions and reflect these in solution set descriptions.*

Sample Tasks:

- The student can solve for θ: $y = \sqrt{\cos(\theta)}$.
- The student can provide domain, range, and graph: $y = \sqrt{\cos(\theta)}$.

4. **Equivalencies:** Successful Trigonometry students are proficient in creating equivalencies in order to simplify expressions, solve equations and inequalities, or take advantage of a common structure or form.

The successful Trigonometry student can:

4a. Purposefully create equivalences and indicate where they are valid.*

Sample Tasks:

- To graph $f(t) = \tan(t)\cos(t)$, the student simplifies to $f(t) = \sin(t)$ and graphs $y = \sin(t)$ with holes at $\left(\frac{\pi}{2} + \mathbb{Z}\pi, \pm 1\right)$.
- To solve $\sqrt{\cos(4t)} = \sqrt{\sin(4t)}$, the student solves $\cos(4t) = \sin(4t)$ and knows this procedure may result in extraneous solutions.

- The student solves $|\cos(2\theta - 3)| + \frac{3}{2} = 2$ by rewriting the left-hand side as a piecewise-defined function.
- The student can rewrite formulas involving multiple occurrences of the variable to formulas involving a single occurrence. Write $a\sin(w\,t) + b\cos(w\,t)$ as $A\sin(w\,t + B)$ or $B\cos(w\,t + B)$.
- The student can rewrite sums as products to reveal attributes such as zeros, envelopes, and phase interference.

4b. Recognize opportunities to create equivalencies in order to simplify workflow.*

Sample Tasks:

- The student recognizes $y = 2\cos(\theta - 3) + 1$ as being related to $y = \cos(\theta)$ via linear transformations and exploits this in order to sketch the graph.
- The student simplifies given trigonometric formulas for graphing reasons.

4c. Become Fluent with conversions using traditional equivalency families.*
[e.g., $(\sin(t))^2 + (\cos(t))^2 = 1$; $(\tan(t))^2 + 1 = (\sec(t))^2$; sums/differences; products; double angle, Euler's Formula ($e^{i\theta} = \cos(\theta) + i\sin(\theta)$), etc.]

Sample Tasks:

- The student can prove trigonometric identities.
- The student solves trigonometric equations.

5. **Modeling with Functions:** Successful Trigonometry students should have experience in using and creating mathematics which models a wide range of phenomena.

The successful Trigonometry student can:

5a. Interpret the function correspondence and behavior of a given model in terms of the context of the model.*

Sample Tasks:

- Given a sales model for lawn chairs, the student can interpret the periodicity, phase shift, and amplitude.
- Given a model of the daily temperature, the student can determine a periodic model and interpret it.
- Given a model of a playground swing's height off the ground, the student can explain the limiting height and where to find corresponding information in the formula.

5b. Create periodic models from data.*

Sample Tasks:

- The student can create formulas for yearly measurements.
- The student can recognize periodic trends in data and create a function to model behavior.

5c. Determine parameters of a model given the form of the model and data.*

Sample Tasks:

- The student can describe effects of changing parameter values for amplitude, phase shift, etc.
- Given the graph of what appears to be a periodic function, the student can use knowledge of the intercepts, minimums/maximums, and asymptotes to create a formula for the function.

5d. Determine a reasonable applied domain for the model, as well as articulate the limitations of the model.*

Sample Task:

- Given the model $h(t) = 6\tan\left(\frac{t}{12}\right)$ which gives height in feet of a model rocket off the ground t seconds after liftoff, the student determines a reasonable applied domain for the model.

6. Appropriate Use of Technology: Successful Trigonometry students are proficient at choosing and applying technology to assist in analyzing functions.

The successful Trigonometry student can:

6a. Anticipate the output from a graphing utility and make adjustments, as needed, in order to efficiently use the technology to solve a problem.*

Sample Tasks:

- The student uses end behavior and a table of values to determine a reasonable window within which to locate the solution to an optimization problem.
- The student can use algebra and technology to produce a detailed graph of $f(\varphi) = \cos\left(\frac{\varphi}{30}\right)\sin(2\varphi - 1)$.

6b. Use technology to verify solutions to equations and inequalities obtained algebraically.*

Sample Task:

- The student solves $|\cos(2t - 3)| + \frac{3}{2} = 2$ and checks the reasonableness of the solution graphically.

6c. Use technology to obtain solutions to equations and inequalities which are difficult to obtain algebraically and know the difference between approximate and exact solutions.*

Sample Tasks:

- The student decides to solve $e^{\sin(x)} = 1 - x$ by graphing $Y1 = e\hat{}(\sin(x))$ and $Y2 = 1 - x$ and observes the solution appears to be $x = 0$. The student then verifies the solution algebraically.
- The student can approximate solutions to $e^{\sin(10x)} = 2 - x^2$.

7. Reasons Mathematically: Successful Trigonometry students demonstrate a proficiency at reasoning mathematically.

The successful Trigonometry student can:

7a. Recognize when a result (theorem) is applicable and use the result to make sound logical conclusions and to provide counter-examples to conjectures.*

Sample Tasks:

- The student recognizes the equation $(\cos(x))^2 - \sin(x) = 5$ as quadratic in form and uses the quadratic formula or factoring to solve for $\sin(x)$. Then the student can determine values of x.
- After identifying forms, the student uses methods from previous courses.
- The student applies characteristics of functions to reason that equations cannot have a solution.
- The student rephrases an equation as information about a function and then identifies solutions based on function analysis.

Additional Learning Outcomes

The OTM Mathematics, Statistics, and Logic Statewide Faculty Review Panel stresses that the essential learning outcomes marked with an asterisk make up the bulk of a Trigonometry course and needs to continue as the focus of this course. Courses that contain only the essential learning outcomes are acceptable from the TMM003 review and approval standpoint. It is up to individual institutions to determine further adaptation of additional course learning outcomes of their choice to support their students' needs. The Statewide Review Panel will not use the additional optional learning outcomes for evaluative purposes and emphasizes that institutions must consider them as optional.

Some institutions expressed an interest in including additional optional learning outcomes to gain such guidance as possibilities to support students' needs. The OTM Mathematics, Statistics, and Logic Statewide Faculty Review Panel developed a few examples in this section, but please know that the additional learning outcomes are absolutely optional and not required. If your institution chooses to explore additional learning outcome(s), the examples below are simply suggestions and not restricted or exhaustive of possibilities. Please note that the samples provided here were crafted as a way to show how to purposefully tie back to the associated required items and to deem as extensions of the essential learning outcomes.

2. **Geometry:** Successful Trigonometry students demonstrate a deep understanding of the measurements of right triangles, right triangles as building blocks of general triangles, and right triangles as a bridge between circular measurements and rectangular measurements.

   The successful Trigonometry student can:

   2d. Describe two-dimensional position using rectangular and polar coordinates, vectors, and parametric equations; demonstrate fluency between any two of these systems; and recognize when one representation would be useful over another in simplifying workflow.

   Sample Tasks:

   - The student can translate between rectangular and polar coordinates.
   - The student can draw curves described by equations involving rectangular and polar coordinates.
   - The student can discuss geometric characteristics of curves defined implicitly: $\sin(x)\cos(y) = \frac{1}{2}$ (level curves).
   - The student can discuss geometric characteristics of curves defined parametrically.
   - The student can discuss the difference between a curve and a parameterization.
   - The student can discuss the difference between a function and a curve.
   - The student successfully finds all points of intersection on the graphs of $r = 4\,cos(2\,\theta)$ and $r = 2$ by understanding that any given point has multiple representations.
   - The student can translate curve descriptions into the language of vectors.
   - The student can discuss direction vectors and lines described parametrically.

2e. Interpret the result of vector computations geometrically and within the confines of a particular applied context (e.g., forces).

Sample Tasks:

- The student can define vectors, their arithmetic, their representation, and interpretations.
- The student can decompose vectors into normal and parallel components.
- The student can interpret the result of a vector computation as a change in location in the plane or as the net force acting on an object.

2f. Represent conic sections algebraically via equations of two variables and graphically by drawing curves.

Sample Tasks:

- The student can perform the process "completing the square" transforming the equation into a standard form.
- The student can draw curves representing conic sections.
- The student can solve systems of equations involving linear and quadratic functions.
- The student can parametrize conic curves.

OHIO TRANSFER MODULE MATHEMATICS, STATISTICS, AND LOGIC: TMM003 TRIGONOMETRY COURSE REVISION FACULTY PARTICIPANTS
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