TMM001: College Algebra (Document produced and finalized, December 8, 2015)

College Algebra is the start of a line of collegiate thought about functions. Functions are the way mathematicians, scientists, and engineers package, describe, and analyze the structure of information. Students need to understand the concept of a function as well as the various tools we use to represent the underlying functional relationship. This line of thought begins with the study of elementary functions and then continues to include more detail through calculus, differential equations, and then application within the student's program of study.

The goal of mathematics in the STEM pathway is to teach students how to use a tool belt full of mathematical tools. It is not enough for students to demonstrate the mechanics of each tool. They need to become critical thinkers, to become fluent in the application of the tools, to use the tools to aid their analysis of a given situation.

- Students need to use function notation to communicate about functional relationships.
- Students need to understand how functional correspondences are encoded in graphs, formulas, and equations.
- Students need to know how to decipher and decode functional correspondences from graphs, formulas, and equations.
- Students need to know how to tie these representations together using each to support the others.
- Students must be able to operating these tools independently in support of an analysis of the functional relationship.

The act of analysis begins with individual pieces and ends with a coherent picture or model. It is never the case that complete answers are simply selected and arranged. Instead multiple descriptions from multiple viewpoints are collected and their equivalence helps us extract properties, characteristics, and structure. This often manifests itself in expressions and equations.

- Students need a richer view of equations than just prompts to solve for a variable.
- Students need to understand equations as descriptions and how to choose and navigate a path through alternatives, options, and equivalencies to move closer to a goal.
- Students need to know algebraic limitations and how estimations also help derive exact solutions.
- Students need to know how use estimations and graphical information to help plan purposeful algebraic strategy.

All of this is tied together in the idea of a model. A model might be a mathematical description of a physical situation. A model might be an equivalent function, expression, or representation of a given mathematical object. In any case, students must be able to direct themselves in an exploration of a model. This might be a correspondence between the mathematics and the physical items. It might be the dissection of a given function. Students need instruction and experience on how to critically think about all of these connections.

While elementary functions are the beginning building blocks, students will eventually encounter many functions that simply cannot be completely described by elementary functions. Technology will be one probe into complicated situations. Students need to begin learning now how technology is used to support functional analysis.

- It isn't enough to operate technological tools.
- It isn't enough to rehearse and perform procedural steps.
- It isn't enough to solve equations for individual variables.
- It isn't enough to evaluate and calculate in isolation.
- It isn't enough to operate detached from meaning.

The current learning environments focused on procedures isn't enough. Students need a conceptual framework in which these procedures are glued together. Students need to develop a thought process that allows them to take the reins of analysis and operate logically on their own. Concepts are not additional work for students. Concepts are the glue that holds everything together and makes it easier for students to learn.

Eventually, students need to move fluently and effectively through a maze of equivalent models of the same information. For example, consider the function modeled in various ways below:

$$F(x) = \frac{1}{x-1} - \frac{1}{x-2}$$

$$F(x) = \frac{-1}{(x-1)(x-2)}$$

$$F(x) = \sum_{n=0}^{\infty} x^n \left(\frac{1}{2^{n+1}} - 1\right)$$

$$F(x) = \frac{4}{1-(2x-3)^2}$$

$$F(x) = \frac{1}{x-1} - \frac{1}{(x-1)-1} = \frac{1}{x-1} + \frac{1}{1-(x-1)}$$

$$F(x) = \sum_{n=-1}^{\infty} (x-1)^n$$

$$F(x) = \int_{3}^{x} \frac{2t-3}{(t-1)^2(t-2)^2}\, dt - \frac{1}{2}$$

$$F(x) = \sum_{n=0}^{\infty} (-1)^n (x-3)^n \left(\frac{1}{2^{n+1}} - 1\right)$$

$$y'(x) + \frac{2x-3}{(x-1)(x-2)}\, y(x) = 0; y(3) = -\frac{1}{2}$$

Students need to be able to choose among the many options and alternatives that best suit their current needs, as well as be able to walk across bridges to more useful representations.

The new viewpoint of the OTM College Algebra guidelines as student outcomes instead of materials presented is simply an attempt to reinforce and better emphasize the larger goal we have always had: to help students think and use rather than rehearse and perform. While we are widening our view and encouraging new effective mathematics courses for our students, we should also be taking the opportunity to rededicate ourselves to the fundamental purpose of STEM education. We want students to think in a particular way. They need direct and deliberate instruction on how to use mathematics to think about STEM topics. We would like our OTM guidelines to provide the same direct and deliberate focus to our faculty.